MATEMATICA

QUESTÃO	RESPOSTA
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Instituto Tecnológico de Aeronáutica

Pró-Reitoria de Pós-Graduação

Prova de Seleção – 2º semestre de 2026 – Questões de Matemática

18 de maio de 2026

Nome do Candidato

Observações

1. Duração da prova: 90 minutos (uma hora e meia)
2. Não é permitido o uso de calculadoras nem softwares nem sites de cálculo numérico e/ou simbólico, bem como não é permitido o uso de IA (inteligência artificial) para auxílio à solução da prova
3. Cada pergunta admite uma única resposta
4. Marque a alternativa que considerar correta no formulário Google enviado por e-mail

Questões em Inglês

1. If $\log_{10}(2) = 0.301$ and $\log_{10}(3) = 0.477$, then $\log_{10}\left(\frac{6\sqrt{2}}{5}\right)$ is

   (a) 0.1295
   (b) 0.2295
   (c) 0.3295
   (d) 0.4295
   (e) 0.5295

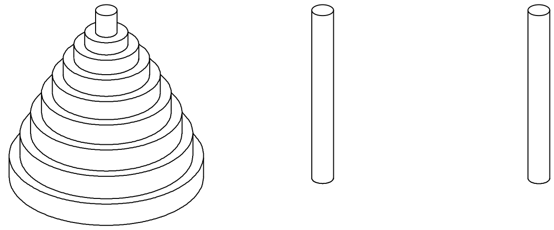

Figure 1: Hanoi Tower with 8 disks.

2. The sum of all solutions of equation $\sin(2x) = \cos(x)$ in the closed interval $[0, 2\pi]$ is
   (a) 3π
   (b) $\frac{7\pi}{2}$
   (c) 4π
   (d) $\frac{9\pi}{2}$
   (e) 5π

3. If the sequence $(-8, a, 22, b, 52)$ is an arithmetic progression, then the product $a \cdot b$ is
   (a) 273
   (b) 259
   (c) 124
   (d) 42
   (e) 15

4. The tower of Hanoi in Figure 1 is a mathematical puzzle consisting of three rods and eight disks of various diameters, which can slide onto any rod. The puzzle begins with the disks stacked on one rod in order of decreasing size, the smallest at the top. The objective of the puzzle is to move the entire stack to one of the other rods, obeying the following rules:
   - Only one disk may be moved at a time.
   - Each move consists of taking the upper disk from one of the stacks and placing it on top of another stack or on an empty rod.
   - No disk may be placed on top of a disk that is smaller than it.

   Under these rules, the number of different distributions of the eight disks in the three rods that can be made is
   (a) $8! \cdot 3!$
   (b) 8^3
   (c) 3^8
   (d) $3 \cdot 8! + 6 \cdot 7! + 6 \cdot 6! \cdot 2! + 6 \cdot 5! \cdot 3! + 6 \cdot 4! \cdot 4!$
   (e) $\frac{(8+3)!}{8! \cdot 3!}$

5. The minimum number of moves required to solve the tower in Figure 1 may be calculated by using recurrence relations as

   (a) 240
   (b) 248
   (c) 250
   (d) 255
   (e) 256

6. A circular ring with radius r is given a uniform electrical charge, the total charge being unitary. The force exerted by the ring on a unit charge x units from the center of the ring and perpendicular to the plane of the ring is

$$F(x) = x\left(x^2 + r^2\right)^{-3/2}.$$

   For $x > 0$, the point at which the force is maximum is

   (a) $x = r$
   (b) $x = r/\sqrt{2}$
   (c) $x = r/\sqrt{3}$
   (d) $x = r/2$
   (e) $x = r/\sqrt{5}$

7. Let x and y be respectively the roots of the quadratic equations $x^2 = 3 + 4i$ and $y^2 = 3 - 4i$, where $i = \sqrt{-1}$. About the sum $x + y$, one can say that

   (a) it may result only one complex number (neither pure real nor pure imaginary).
   (b) it may result only two complex numbers (neither pure real nor pure imaginary).
   (c) it may result two real and two imaginary numbers, all of them with integer parts.
   (d) it may result four complex numbers with irrational real and complex parts.
   (e) it may result four complex numbers with non-integer rational real and complex parts.

8. Let

$$x = \frac{1}{2} + \frac{1}{6} + \frac{1}{12} \cdots = \sum_{j=1}^{\infty} \frac{1}{j(j+1)}$$

   The value of x is

   (a) $\frac{3}{4}$
   (b) $\frac{4}{5}$
   (c) 1
   (d) $\frac{\pi^2}{6}$
   (e) ∞

9. Let $a, b \in \mathbb{N}$ with $a > b$. x and y are defined as

$$\begin{aligned} x &= \sqrt{(a^2+b^2)^2 - (a^2-b^2)^2} \\ y &= \sqrt{(a^2-b^2)^2 - 4a^2b^2} \end{aligned}$$

One can say that

(a) Both x and y may not be integer for some values of a and b.

(b) x will always be integer, while y may not be integer for some values of a and b.

(c) y will always be integer, while x may not be integer for some values of a and b.

(d) x and y will be integer for any value of a and b.

(e) x and y may be non-integer rational numbers.

10. In the first part of a journey, a car takes a 20 km long dirt road. After that, it takes a 40 km long highway, driving at a speed of 120 km/h. If the average speed of the entire travel is of 60 km/h, the speed developed in the dirt road is

(a) 15 km/h

(b) 20 km /h

(c) 25 km/h

(d) 30 km/h

(e) 40 km/h

11. About the function

$$f(x) = a\left(4 - x^2\right) + (1-a)\left(6 - 4x + x^2\right)$$

one *cannot say* that

(a) $f(1) = 3$ for any value of a

(b) $y = 5 - 2x$ is tangent to $f(x)$ only at $x = 1$ for any value of $a \neq \frac{1}{2}$

(c) $f(x)$ will have only one real root for $a = \frac{1}{2}$

(d) $f(x)$ will have a double root for $a = \frac{1}{3}$

(e) $f(x)$ will have at least one real root for any real value of a

12. The value of the integral $\int_1^2 \frac{[\ln(x)]^2}{x} dx$ is

(a) $\dfrac{2^3}{3}$

(b) $\dfrac{1}{3 \cdot 2^3}$

(c) $\dfrac{[\ln(2)]^3}{3}$

(d) $\dfrac{1}{[\ln(2)]^3 3}$

(e) ∞

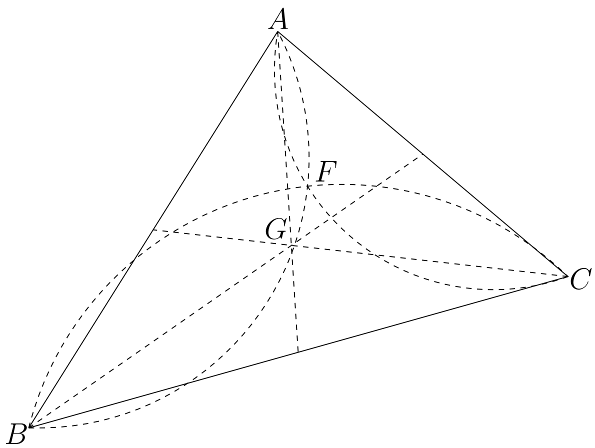

Figure 2: A triangle with its centroid or barycenter G and its Fermat point F.

13. About the system of equations

$$\begin{cases} (x-1)^2 + y^2 + z^2 = 1 \\ x^2 + y^2 - z^2 = 0 \\ x - z^2 = 0 \end{cases}$$

one can say that

(a) it has no real solutions.
(b) it has only one real solution.
(c) it has only two distinct real solutions.
(d) it has only four distinct real solutions
(e) it has several (infinite) distinct solutions.

14. In Figure 2, F is the *Fermat Point* of triangle ABC ($\widehat{AFB} = \widehat{BFC} = \widehat{CFA} = 120°$) while G is its centroid (intersection of its medians). Take the following definitions:

- H is the point with the minimum sum of distances $\overline{HA} + \overline{HB} + \overline{HC}$
- I is the point with the minimum value for $\sqrt{\overline{IA}^2 + \overline{IB}^2 + \overline{IC}^2}$

Mark the correct choice:

(a) $F = H$ and $G = I$
(b) $F = I$ and $G = H$
(c) $F = H$ but $G \neq I$
(d) $F \neq H$ and $G = I$
(e) $F \neq H$, $G \neq I$, $F \neq I$ and $G \neq H$

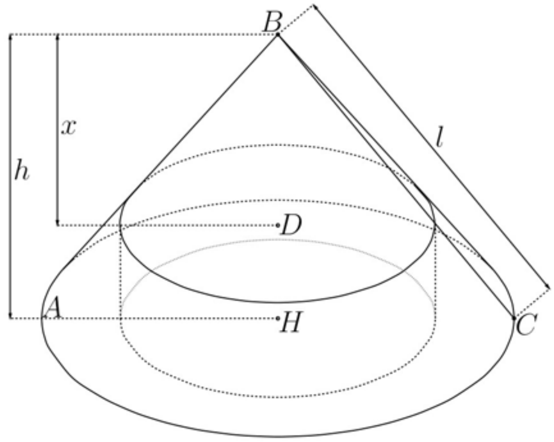

Figure 3: Cone with inscribed cylinder.

15. In Figure 3, $h = \overline{BH}$ is the height of a cone, $l = \overline{BC}$ is its slant height (*geratriz* in portuguese) and $x = \overline{BD}$ is the difference between the height of the cone and the height of a cylinder that is inscribed in it. In order for the lateral surface of the cone above the cylinder to be equal to the lateral surface of the cylinder, it is necessary that

   (a) $x = \dfrac{h^2}{l}$

   (b) $x = \dfrac{l^2}{h}$

   (c) $x = \dfrac{2l^2}{2l + h}$

   (d) $x = \dfrac{2h^2}{l + 2h}$

   (e) $x = \dfrac{h^2 + l^2}{2l}$

16. Let $A = \{x | x \in \mathbb{N}, 1 \leq x \leq 100\}$. What is the number of elements of the subset

   $B = \{x | x \in A, x$ is not divisible by 3, x is not divisible by 5 and x is not divisible by $7\}$?

   (a) 33

   (b) 36

   (c) 40

   (d) 42

   (e) 45