

1	d
2	c
3	b
4	c
5	a
6	d
7	d
8	c
9	b
10	d
11	e
12	b
13	b
14	b
15	b
16	a



Instituto Tecnológico de Aeronáutica

Pró-Reitoria de Pós-Graduação

Prova de Seleção – 1º semestre de 2026 – Questões de Matemática

03 de novembro de 2025

Nome do Candidato

Observações

1. Duração da prova: 90 minutos (uma hora e meia)
2. Não é permitido o uso de calculadoras nem softwares nem sites de cálculo numérico e/ou simbólico, bem como não é permitido o uso de IA (inteligência artificial) para auxílio à solução da prova
3. Cada pergunta admite uma única resposta
4. Marque a alternativa que considerar correta no formulário Google enviado por e-mail

Questões em Inglês

1. About the system of equations

$$\begin{cases} x + y = 0 \\ \frac{y}{x} = -1 \end{cases},$$

one can say that

- (a) It has no real solution
- (b) It has just one real solution
- (c) It has exactly two distinct real solutions
- (d) It has an infinite number of distinct real solutions
- (e) It is not possible to discuss this system and more information is required

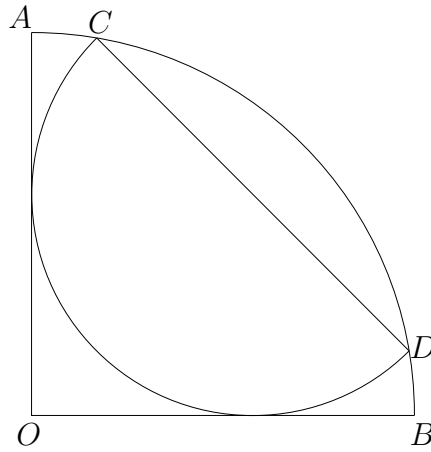


Figure 1: Semicircle tangent to a quadrant

2. In a building , the upper water reservoir may be supplied by any of three water pumps, each of them having its independent piping system. In a pumping test, each of them was used separately to fill the upper reservoir, which was initially empty. These were the results:

- I) Pump A took one hour to fill the upper water reservoir.
- II) Pump B took 1.5 hours to fill the upper water reservoir.
- III) Pump C took 3 hours to fill the upper water reservoir.

If now the upper reservoir is empty and the three pumps are working simultaneously, how many minutes will they take to fill the reservoir?

- (a) 15 minutes
 - (b) 20 minutes
 - (c) 30 minutes
 - (d) 45 minutes
 - (e) 60 minutes
3. In Figure 1, AOB is a quadrant of a circle and semicircle \widehat{CD} is tangent to the lines OA and OB . The ratio between the areas of the quadrant AOB and the semicircle \widehat{CD} is
- (a) 4 : 3
 - (b) 3 : 2
 - (c) $\pi : 2$
 - (d) $\sqrt{2} : 1$
 - (e) 2 : 1

4. The equation

$$4^x - 2^{x+2} + 3 = 0$$

- (a) has no real solution.
- (b) has one real solution
- (c) has two real solutions
- (d) has three real solutions
- (e) has four real solutions

5. A cube should have two of its faces painted in red, while the other ones should be painted in other four different colors. Two painting schemes are considered equal if one can get all faces of the same colors in the same direction by simple rotations. How many different painting schemes can be formed for this cube?

- (a) 15 different schemes
- (b) 16 different schemes
- (c) 20 different schemes
- (d) 24 different schemes
- (e) 30 different schemes

6. If $C_n^k = \frac{n!}{(n-k)!k!}$, the largest value of $C_{32}^p 2^{32-p} \left(\frac{1}{2}\right)^p$ is obtained for

- (a) $p = 5$
- (b) $p = 32$
- (c) $p = 24$
- (d) $p = 6$
- (e) $p = 16$

7. The value of the expression

$$4(1 + \sqrt{3}) - 6(1 + \sqrt{3})^2 + 4(1 + \sqrt{3})^3 - (1 + \sqrt{3})^4$$

is

- (a) 9
- (b) 1
- (c) $-4\sqrt{3}$
- (d) -8
- (e) $-2\sqrt{27}$

8. Consider a curve given in polar coordinates by

$$r = \theta$$

for $\theta \geq 0$, where $\theta = 0$ radians is the direction of the x axis in the Cartesian coordinate system. About this curve in Cartesian coordinates, two statements are given:

I) In Cartesian coordinates, the derivative of this curve is

$$\frac{dy}{dx} = \frac{\cos(\theta) - \theta \sin(\theta)}{\sin(\theta) + \theta \cos(\theta)}$$

II) A point in which the tangent to the curve is parallel to the y axis of the Cartesian coordinate system is where $\theta \tan(\theta) = 1$.

Mark the correct option:

- (a) Both statements (I) and (II) are wrong.
- (b) Statement (I) is correct, but statement (II) is wrong.
- (c) Statement (I) is wrong, but statement (II) is correct.
- (d) Both statements (I) and (II) are correct.
- (e) Nothing can be said and more information is necessary.

9. The expression

$$\cos^3\left(\frac{\pi}{3}\right) - 3i \cos^2\left(\frac{\pi}{3}\right) \sin\left(\frac{\pi}{3}\right) - 3 \cos\left(\frac{\pi}{3}\right) \sin^2\left(\frac{\pi}{3}\right) + i \sin^3\left(\frac{\pi}{3}\right)$$

may be simplified as

- (a) $\cos\left(\frac{\pi}{2}\right) + i \sin\left(\frac{\pi}{2}\right)$
- (b) $\cos(-\pi) + i \sin(-\pi)$
- (c) $\cos\left(\frac{4\pi}{3}\right) + i \sin\left(\frac{4\pi}{3}\right)$
- (d) $\cos\left(\frac{-4\pi}{3}\right) + i \sin\left(\frac{-4\pi}{3}\right)$
- (e) $\cos\left(\frac{5\pi}{3}\right) + i \sin\left(\frac{5\pi}{3}\right)$

10. Figure 2 shows a plot of the function $y(x) = \log_{10}(x)$. If $OA = BC$, then

- (a) $\log_a(b) = c$
- (b) $a + b = c$
- (c) $a^c = b$
- (d) $ab = c$
- (e) $10^a + 10^b = 10^c$

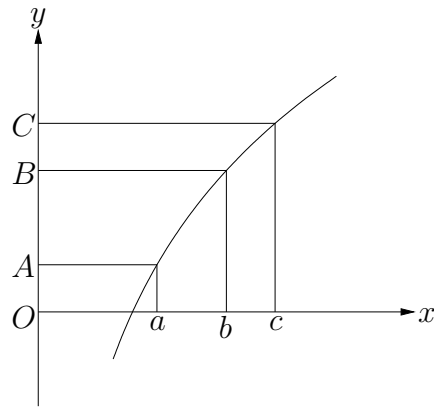


Figure 2: Plot of the function $y(x) = \log_{10}(x)$

11. Consider the 2×2 matrix equation

$$[X]^2 - 2[X] + 2[I] = [0],$$

in which $[X]$ is an unknown matrix, $[I]$ is the identity matrix and $[0]$ is the null matrix. Mark the option which is not a root of the equation above:

- (a) $\begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix}$
- (b) $\begin{bmatrix} 1 & -i \\ -i & 1 \end{bmatrix}$
- (c) $\begin{bmatrix} 1 & i \\ i & 1 \end{bmatrix}$
- (d) $\begin{bmatrix} 1+i & 0 \\ 0 & 1-i \end{bmatrix}$
- (e) $\begin{bmatrix} i & 1 \\ 1 & i \end{bmatrix}$

12. The value of the sum

$$S = 1 + \frac{1}{2}i - \frac{1}{4} - \frac{1}{8}i + \frac{1}{16} + \dots$$

is

- (a) $\frac{4}{5} - \frac{2}{5}i$
- (b) $\frac{4}{5} + \frac{2}{5}i$
- (c) $1 - \frac{1}{2}i$
- (d) $1 + \frac{1}{2}i$
- (e) $\frac{4}{3} + \frac{2}{3}i$

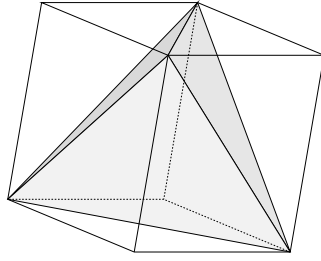


Figure 3: Tetrahedron inscribed in a cube

13. The value of

$$\int_0^1 \sqrt[3]{x \sqrt{x \sqrt{x \sqrt{x \sqrt{x \dots}}}}} dx$$

is

- (a) $\frac{3}{5}$
- (b) $\frac{2}{3}$
- (c) 1
- (d) $\frac{3}{2}$
- (e) $\frac{5}{3}$

14. If $1 + 2 + 3 + \dots + n = 105$, then n is

- (a) 12
- (b) 14
- (c) 11
- (d) 13
- (e) 15

15. In Figure 3, the ratio of the volume of the tetrahedron to the volume of the cube is

- (a) $\frac{1}{4}$
- (b) $\frac{1}{3}$
- (c) $\frac{\sqrt{3}}{4}$
- (d) $\frac{1}{2}$
- (e) $\frac{2}{3}$

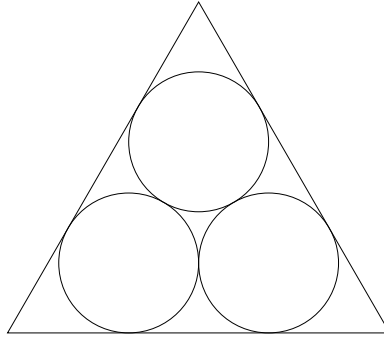


Figure 4: Triangle with three circles

16. In Figure 4, three circles have radii R and are inscribed in an equilateral triangle with side length of 2 cm. The value of R is

- (a) $\frac{1}{1 + \sqrt{3}}$ cm
- (b) $\frac{\sqrt{3}}{1 + \sqrt{3}}$ cm
- (c) $\frac{3}{1 + \sqrt{2}}$ cm
- (d) $\frac{\sqrt{3}}{2 + \sqrt{3}}$ cm
- (e) $\frac{2}{2 + \sqrt{3}}$ cm