1	d
2	d
3	c
4	b
5	c
6	c
7	b
8	c
9	c
10	d
11	a
12	d
13	a
14	c
15	a
16	e

Instituto Tecnológico de Aeronáutica

Pró-Reitoria de Pós-Graduação

Prova de Seleção – 2º semestre de 2025 – Questões de Matemática

19 de maio de 2025

Nome do Candidato

Observações

1. Duração da prova: 90 minutos (uma hora e meia)
2. Não é permitido o uso de calculadoras nem softwares nem sites de cálculo numérico e/ou simbólico, bem como não é permitido o uso de IA (inteligência artificial) para auxílio à solução da prova
3. Cada pergunta admite uma única resposta
4. Marque a alternativa que considerar correta no formulário Google enviado por e-mail

Questões em Inglês

1. The value of the limit

$$\lim_{x \to 2} \frac{\sqrt{3x-2}-2}{\sqrt{4x+1}-3}$$

is

(a) 1/2
(b) 2/3
(c) 3/4
(d) 9/8
(e) 4/3

2. $\sqrt{19+6\sqrt{10}}-\sqrt{19-6\sqrt{10}}$ is equal to

   (a) 3

   (b) 4

   (c) 5

   (d) 6

   (e) $2\sqrt{10}$

3. Let $f(x)=(x-x_1)^2(x-x_2)^2$, where $x_1=\frac{1+\sqrt{5}}{2}$ and $x_2=\frac{1-\sqrt{5}}{2}$. Mark the *wrong* statement about $f(x)$:

   (a) $f(x)=x^4-2x^3-x^2+2x+1$

   (b) $f(x)=(x^2-x-1)^2$

   (c) $f(x)=(x+2)(x+1)x(x-1)+1$

   (d) x_1 and x_2 are roots of the derivative $f'(x)$

   (e) $x=1/2$ is a root of the derivative $f'(x)$

4. The value of the sum

$$\binom{20}{0}+\binom{20}{1}2+\binom{20}{2}2^2+\cdots+\binom{20}{20}2^{20}$$

   is

   (a) 2^{40}

   (b) 9^{10}

   (c) 20^{20}

   (d) $20!$

   (e) $40!/20!$

5. In geometry, a *space diagonal* (also *interior diagonal* or *body diagonal*) of a polyhedron is a line connecting two vertices that are not on the same face. Space diagonals contrast with face diagonals, which connect vertices on the same face (but not on the same edge). Figure 1 shows a dodecahedron with its edges in background as dashed segments, its edges in foreground as solid lines and just one of its space diagonals as a dash-dotted line. How many distinct space diagonals does the regular dodecahedron have?

   (a) 36

   (b) 72

   (c) 100

   (d) 120

   (e) 160

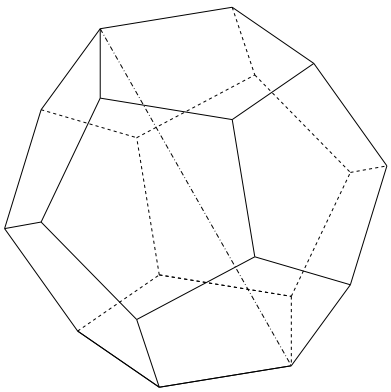

Figure 1: A dodecahedron with one of its space diagonals

6. Two equations involving two unknown variables x and y are given:

   I. $\dfrac{xy}{x-y} = 5$

   II. $\dfrac{xy}{x+y} = 1$

   If it is asked to determine the value of $\dfrac{x^2-y^2}{x^2y^2} + \dfrac{2}{xy}$, one can say that

   (a) equation I *alone* is sufficient, but equation II *alone* is *not* sufficient to determine the expression given;

   (b) equation II *alone* is sufficient, but equation I *alone* is *not* sufficient to determine the expression given;

   (c) *both* equations I and II *together* are sufficient to determine the expression given, but *neither* equation *alone* is sufficient;

   (d) *each* equation *alone* is sufficient to determine the expression given;

   (e) equations I and II *together* are *not* sufficient to determine the expression given, and additional data specific to the problem are needed.

7. If x is a positive real number with $x \neq 1$ and $x \neq \dfrac{1}{3}$ and x satisfies the equation

$$\frac{2+\log_3(x)}{\log_{x+2}(x)} - \frac{\log_x(x+2)}{1+\log_3(x)} = \log_x(x+2),$$

   then x belongs to the interval

   (a) $\left(0, \dfrac{1}{81}\right)$

   (b) $\left(\dfrac{1}{81}, \dfrac{1}{3}\right)$

   (c) $\left(\dfrac{1}{2}, 1\right)$

   (d) $\left(1, \dfrac{3}{2}\right)$

   (e) $\left(\dfrac{3}{2}, 2\right)$

8. About complex numbers in general ($z = x + y\,i$, where x and y are real numbers and $i = \sqrt{-1}$), two statements are given:

   I. For any $z = x + y\,i \in \mathbb{C}, z^2 = (\bar{z})^2$, where $\bar{z} = x - y\,i$ is the complex conjugate of z

   II. For any $z = x + y\,i \in \mathbb{C}, \overline{z^4} = (\bar{z})^4$

   Mark the correct option about these satements:

   (a) both statements I and II are false;

   (b) statement I is true, but statement II is false;

   (c) statement I is false, but statement II is true;

   (d) both statements I and II are true;

   (e) statement II is an unsolved problem for complex numbers.

9. Three professional craftsmen make 24 craft pieces in 2 hours, while 4 apprentice craftsmen make 16 craft pieces in 3 hours. How many hours should 2 professionals and 3 apprentices take in order to make 48 craft pieces?

   (a) 2

   (b) 3

   (c) 4

   (d) 5

   (e) 6

10. Given the real matrix

$$A = \begin{bmatrix} a & c & e & e \\ 0 & a & e & e \\ 0 & 0 & b & d \\ 0 & 0 & 0 & b \end{bmatrix},$$

    which option would ensure that $\det(A) > 0$?

    (a) $c^4de > 0$

    (b) $a + b > 0$

    (c) $a^2 + b^2 > 0$

    (d) $ab \neq 0$

    (e) $cde \neq 0$

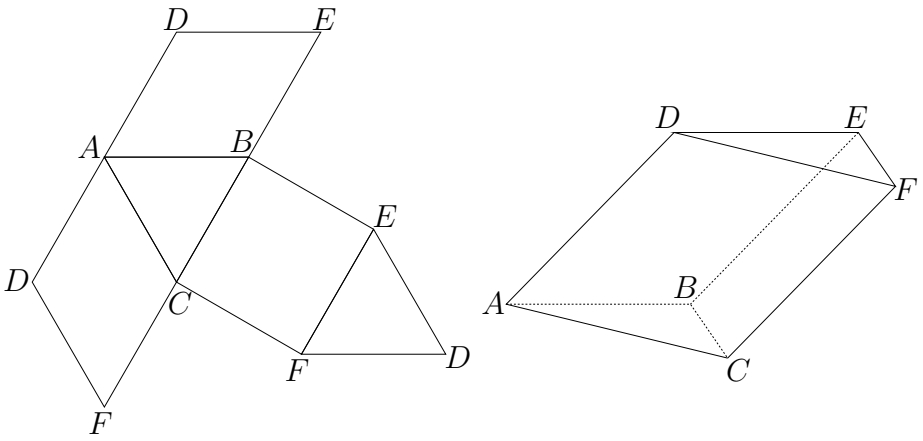

Figure 2: Net of a prismatic polyhedron and its perspective

11. In geometry, a *net of a polyhedron* is an arrangement of non-overlapping edge-joined polygons in the plane which can be folded (along edges) to become the faces of the polyhedron. Figure 2 shows the net of a prismatic polyhedron. There, all the edges have unit length. Moreover, diagonals BD of the face $ABED$ and CD of the face $ACFD$ also have unit length. What is the volume of the folded polyhedron?

    (a) $\sqrt{2}/4$

    (b) $3/8$

    (c) $\sqrt{3}/4$

    (d) $1/2$

    (e) The polygons presented in the plane cannot be folded into a closed polyhedron

12. How many integers n greater than 10 and less than 100 are there such that, if the digits of n are reversed, the resulting integer is $n + 9$?

    (a) 5

    (b) 6

    (c) 7

    (d) 8

    (e) 9

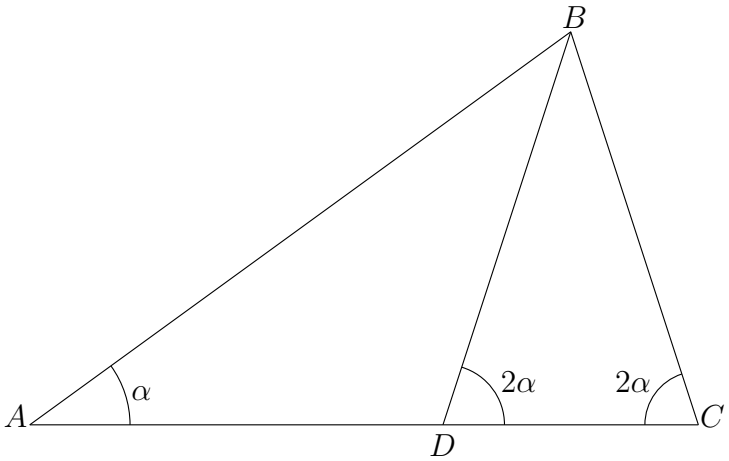

Figure 3: Triangle with a cevian

13. About the triangle of Figure 3, two statements are given:

    I. Line segment AD has length 6

    II. $\alpha = 36°$

    On data sufficiency to numerically determine the length of BC, mark the correct choice:

    (a) Statement I *alone* is sufficient, but statement II is *not sufficient* to determine it;

    (b) Statement II *alone* is sufficient, but statement I is *not sufficient* to determine it;

    (c) *Both* statements I and II *together* are sufficient to answer the question asked, but *neither* statement *alone* is sufficient;

    (d) *Each* statement *alone* is sufficient to answer the question asked;

    (e) Statements I and II *together* are *not* sufficient to answer the question asked, and additional data specific to the problem are needed.

14. In a mayoral election, Candidate A received 1/3 more votes than Candidate B, and Candidate B received 1/4 fewer votes than Candidate C. If Candidate C received 12000 votes, how many votes did Candidate A receive?

    (a) 9000

    (b) 11000

    (c) 12000

    (d) 13000

    (e) 16000

15. If a, b, and m are positive and $\dfrac{m+a}{m+b} = \dfrac{3a}{b}$, what is the value of m?

(a) $\dfrac{2ab}{b-3a}$

(b) $\dfrac{b}{3a}$

(c) $\dfrac{a}{2b}$

(d) $\dfrac{4ab}{b-3a}$

(e) $\dfrac{b-3a}{2ab}$

16. Four married couples (four men, each of them married to a woman) take part of a dance in which the pairs with a man and a woman are formed by chance. What is the probability that each man dance with a woman other than his wife?

(a) 1/4

(b) 5/18

(c) 5/16

(d) 1/3

(e) 3/8