2025-1																
Questão	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
Resposta	a	c	c	e	b	e	b	a	b	b	e	c	b	e	c	d

Instituto Tecnológico de Aeronáutica

Pró-Reitoria de Pós-Graduação

Prova de Seleção – 1º semestre de 2025 – Questões de Matemática

11 de novembro de 2024

Nome do Candidato

Observações

1. Duração da prova: 90 minutos (uma hora e meia)
2. Não é permitido o uso de calculadoras nem softwares nem sites de cálculo numérico e/ou simbólico, bem como não é permitido o uso de IA (inteligência artificial) para auxílio à solução da prova
3. Cada pergunta admite uma única resposta
4. Marque a alternativa que considerar correta no formulário Google enviado por e-mail

Questões em Inglês

1. Six distinct lines are tangent to a circle and they are not parallel to each other. How many distinct regions are formed by these lines?

   Observation: In this problem, a region is defined as a set of points where none of the tangent lines defined above intersects the segment connecting any two points of this set; hence, the whole circle belongs to one of these regions defined above.

   (a) 22 regions
   (b) 24 regions
   (c) 32 regions
   (d) 48 regions
   (e) 64 regions

2. In order to know if the system

$$\begin{cases} a\,x + b\,y + c\,z = g \\ d\,y + e\,z = h \\ f\,z = i \end{cases}$$

has only one unique solution in x, y and z, one should know the value of the product

(a) $a\,b\,c$

(b) $a\,b\,d$

(c) $a\,d\,f$

(d) $a\,e\,f$

(e) $g\,h\,i$

3. The development of $\left(x + \frac{1}{x^2}\right)^n$ has a term independent of x if

(a) n is even

(b) n is odd

(c) n is divisible by 3

(d) n is any number different from zero

(e) there is no value of n that satisfies this condition

4. In how many ways the letters of *vestibular* can be arranged such that the letters *ves* appear together, in any order?

(a) 306040

(b) 404060

(c) 200020

(d) 120260

(e) 241920

5. About the function

$$f(x) = \arcsin\left(\arcsin\left(x\right)\right),$$

mark the *wrong* statement:

(a) The *real* function $f(x)$ is bijective wth the same image of $\arcsin(x)$.

(b) The *real* function $f(x)$ has the *open* interval $(-1, 1)$ as its domain.

(c) The inverse real function $f^{-1}(x)$ is an odd function in its domain.

(d) The derivative of this function is $\dfrac{\mathrm{d}}{\mathrm{d}x} f(x) = \dfrac{1}{\sqrt{\left(1 - x^2\right)\left(1 - \arcsin^2\left(x\right)\right)}}$.

(e) The derivative of the inverse function $f^{-1}(x)$ is $\dfrac{\mathrm{d}}{\mathrm{d}y} f^{-1}(y) = \cos\left(\sin\left(y\right)\right)\cos\left(y\right)$

6. The calculus of the determinant $\begin{vmatrix} 1 & \ln(2) & \ln(4) \\ -1 & \ln(4) & \ln(8) \\ 1 & \ln(8) & \ln(16) \end{vmatrix}$ results in

   (a) 0

   (b) 1

   (c) $-4\ln(2)$

   (d) $-8\ln(2)$

   (e) $-4\ln^2(2)$

7. About the polynomial equation

$$x^3 + x - 10 = 0$$

   one can say that

   (a) It has no real roots

   (b) It has one real rational root

   (c) It has one real irrational root

   (d) It has three real roots, all of them are integer

   (e) It has three real roots, all of them are irrational

8. There are 4 card-processing machines in an office. The fastest of these machines processes x cards in 7 hours and the slowest processes x cards in 8 hours. Which of the following could *not* be the average time per machine for each of the 4 machines to process x cards?

   (a) 7.2

   (b) 7.3

   (c) 7.5

   (d) 7.6

   (e) 7.7

9. The system of equations

$$\begin{cases} \dfrac{1}{1-x} = \dfrac{1}{1-\dfrac{1}{x}} \\ x + y = 1 \end{cases}$$

   (a) has no real solution.

   (b) has one real solution.

   (c) has two real solutions.

   (d) has several real solutions.

   (e) cannot be discussed as the first equation does not contain y.

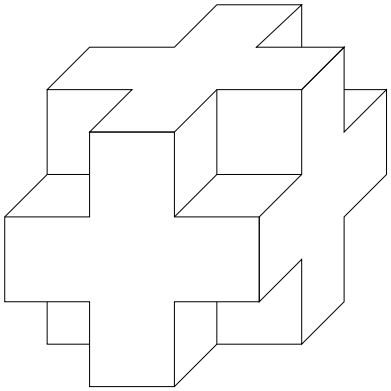

Figure 1: Cube truncated in the vertices

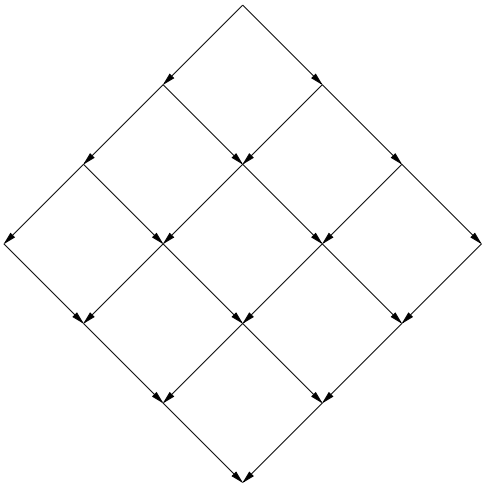

Figure 2: Paths downward over a square

10. In Figure 1, from a cube with edge length of l, cubes having edge length of $\frac{l}{3}$ were removed from all vertices. Mark the *wrong* statement about the solid resulting from this operation:

    (a) The resulting polyhedron has 30 faces.

    (b) The resulting solid has 54 vertices.

    (c) The resulting solid has 84 edges.

    (d) The resulting polyhedron surface has an area equal to $6l^2$.

    (e) The resulting polyhedron has a volume equal to $\frac{19}{27}l^3$.

11. Figure 2 shows a square over which oriented paths lead from the highest vertex to the lowest one. How many different paths from the highest vertex to the lowest one can be taken over the square by following the arrows indicated?

    (a) 8

    (b) 12

    (c) 16

    (d) 18

    (e) 20

12. There are six color inks and each one is to be used to paint a face of a cube. Considering that two coloring schemes are equal if all the sides of equal colors can be aligned with a single three-dimensional rotation, how many different coloring schemes can be defined for this cube?

    (a) 10

    (b) 20

    (c) 30

    (d) 40

    (e) 60

13. A region in the Cartesian space is delimited by the planes $z = 0$, $z = 1$ and the surface of revolution around the z-axis of the straight line connecting the points $(1, 0, 0)$ and $(0, 1, 1)$. What is the volume of this region?

    (a) $\frac{1}{2}\pi$

    (b) $\frac{2}{3}\pi$

    (c) $\frac{\sqrt{2}}{2}\pi$

    (d) $\frac{3}{4}\pi$

    (e) π

14. Which of these expressions produces the same value of $\sqrt[3]{5\sqrt{2} - 7}$?

    (a) $\sqrt{6 - 4\sqrt{2}}$

    (b) $5\sqrt[2]{2} - 6$

    (c) $\sqrt[6]{50} - \sqrt[3]{3}$

    (d) $\sqrt[6]{99 + 40\sqrt{2}}$

    (e) $\frac{1}{\sqrt{2} + 1}$

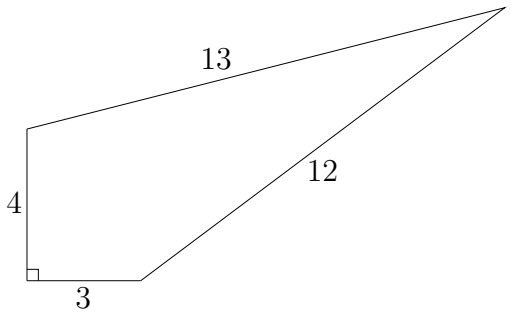

Figure 3: Quadrilateral with right angle

15. Figure 3 shows a quadrilateral with an internal right angle. About this quadrilateral, two statements are posed:

    I. The internal area of this quadrilateral is 36.

    II. This quadrilateral can be inscribed in a circle.

    Mark the correct option:

    (a) Statements I and II are wrong.

    (b) Statement I is wrong while statement II is true.

    (c) Statement I is true while statement II is false.

    (d) Both statements I and II are true

    (e) The information in Figure 3 is not enough to determine this quadrilateral properties, as only one angle is given.

16. Given the equations of two lines,

$$\begin{aligned} x + y &= \frac{\pi}{2} \\ y &= \frac{1}{2}\left(\sqrt{3} + \frac{\pi}{6} - x\right), \end{aligned}$$

    mark the *wrong* statement about them:

    (a) The first line is tangent to curve $y = \cos(x)$

    (b) The first line has only one point in common with curve $y = \cos(x)$

    (c) The second line is tangent to curve $y = \cos(x)$

    (d) The second line has only one point in common with curve $y = \cos(x)$

    (e) The intersection of these lines occurs at $y = \sqrt{3} - \frac{\pi}{3}$