2024-2																
Questão	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
Resposta	b	c	c	e	c	e	d	c	d	b	e	e	e	a	c	d

Instituto Tecnológico de Aeronáutica

Pró-Reitoria de Pós-Graduação

Prova de Seleção – 2º semestre de 2024 – Questões de Matemática

20 de maio de 2024

Nome do Candidato

Observações

1. Duração da prova: 90 minutos (uma hora e meia)
2. Não é permitido o uso de calculadoras nem softwares nem sites de cálculo numérico e/ou simbólico
3. Cada pergunta admite uma única resposta
4. Marque a alternativa que considerar correta no formulário Google enviado por e-mail

Questões em Inglês

1. In a certain season, a river flows to a reservoir while its floodgates are open. In this condition, it takes 24 days to dump all the water of the full reservoir. If the reservoir is empty and the floodgates are closed, it takes 12 days to fill the reservoir. If the reservoir is full, the floodgates are open but the river is diverted from the reservoir, how many days will be necessary to dump all the water from the reservoir?

   (a) 6 days
   (b) 8 days
   (c) 12 days
   (d) 15 days
   (e) 18 days

2. The function

$$f(x) \;=\; x\ln\left(\frac{1}{x}\right)$$

is well defined in the real interval $(0,1]$. About this function, one *cannot* say that

(a) $\lim_{x\to 0} f(x) = 0$

(b) $f(1) = 0$

(c) $\dfrac{\mathrm{d}f}{\mathrm{d}x}(x) = 1 + \ln\left(\dfrac{1}{x}\right)$

(d) $f(x)$ has a maximum value at $x = e^{-1}$, where e is base of natural logarithms

(e) $\displaystyle\int f(x)\mathrm{d}x = \frac{1}{4}x^2\left[1 - 2\ln(x)\right] + C$, where C is a (real) constant of integration

3. Let the function $f(x)$ be defined in the interval $[x_0, x_1]$. This function is such that

- $\dfrac{\mathrm{d}^2 f}{\mathrm{d}x^2}(x) = a \neq 0, \forall x \in [x_0, x_1]$
- $\dfrac{\mathrm{d}f}{\mathrm{d}x}(x_0) = b$
- $f(x_0) = c$

Two statements are posed about this function:

(I) $f(x) = a\left(x - x_0\right)^2 + b\left(x - x_0\right) + c$

(II) $\left[\dfrac{\mathrm{d}f}{\mathrm{d}x}\left(x_1\right)\right]^2 = \left[\dfrac{\mathrm{d}f}{\mathrm{d}x}\left(x_0\right)\right]^2 + 2a\left[f\left(x_1\right) - f\left(x_0\right)\right]$

Mark the right option about these statements

(a) Both statements (I) and (II) are correct

(b) Statement (I) is correct but statement (II) is wrong

(c) Statement (II) is correct but statement (I) is wrong

(d) Both statements (I) and (II) are wrong

(e) More information is needed to analyze the veracity of these statements

4. A path along 5 edges of a tetrahedron can be described by the enumeration of the six labels of the vertices to be followed, presented in the order to be followed. No path should pass through any edge more than one time. For example, the path $1 \to 2 \to 3 \to 4 \to 1 \to 3$ is depicted in Figure 1. How many different proper paths can be formed along 5 of the six edges of a tetrahedron?

(a) 24

(b) 36

(c) 48

(d) 60

(e) 72

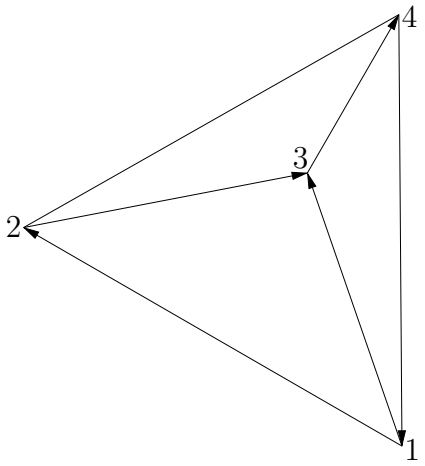

Figure 1: A path $1 \to 2 \to 3 \to 4 \to 1 \to 3$ drawn along numbered edges of a tetrahedron

5. About the system of equations

$$\begin{cases} x^2 - xy + y^2 = 7 \\ x^3 + y^3 = 35 \end{cases},$$

How many solutions does it have?

(a) No real solutions

(b) 1 real solution

(c) 2 real solutions

(d) 3 real solutions

(e) 6 real solutions

6. Put in simple form, an *eigenvector* of a matrix $[M]$ is a vector $\{v\}$ that preserves its direction when multiplied by the matrix, only eventually changing its magnitude or orientation, depending on the value of the eigenvalue associated to it, i.e.,

$$[M]\{v\} = \lambda\{v\}.$$

In the above equation, λ is the *eigenvalue* associated to the eigenvector $\{v\}$ of matrix $[M]$. Given the matrix

$$M = \begin{bmatrix} 1 & -0.3 & 0 \\ -0.3 & 1 & 0 \\ 0 & 0 & 1.3 \end{bmatrix},$$

mark the option which is *not* an eigenvector of $[M]$:

(a) $\{\sqrt{2}/2, -\sqrt{2}/2, 0\}^T$

(b) $\{0, 0, 1\}^T$

(c) $\{\sqrt{3}/3, -\sqrt{3}/3, \sqrt{3}/3\}^T$

(d) $\{1, 1, 0\}^T$

(e) $\{\sqrt{3}/3, \sqrt{3}/3, \sqrt{3}/3\}^T$

7. How many equal spheres can be stacked in order to form a pyramid having a square basis with six spheres in each edge?

   (a) 48 spheres
   (b) 56 spheres
   (c) 72 spheres
   (d) 91 spheres
   (e) 96 spheres

8. If $i = \sqrt{-1}$, i^i has the value of

   (a) 1
   (b) -1
   (c) $\exp(-\pi/2 + 2k\pi)$, with $k \in \mathbb{Z}$
   (d) $i + 1$
   (e) $\sinh(-1)$

9. About *hyperbolic functions*, the following statements are posed:

   - $\sinh(a+b) = \sinh(a)\cosh(b) + \sinh(b)\cosh(a)$
   - $\cosh(a+b) = \cosh(a)\cosh(b) - \sinh(b)\sinh(a)$
   - $\sinh(3a) = 3\sinh(a)\cosh^2(a) + \sinh^3(a)$
   - $\cosh(3a) = 3\sinh^2(a)\cosh(a) + \cosh^3(a)$

   Mark the correct option:

   (a) All statements are wrong
   (b) There is only one correct statement
   (c) There are just two correct statements
   (d) There is only one false statement
   (e) All statements are true

10. The sequence given by

$$A_n = \frac{1}{\sqrt{3}i}\left[\left(\frac{1+\sqrt{3}i}{2}\right)^n - \left(\frac{1-\sqrt{3}i}{2}\right)^n\right], n \in \mathbb{N},$$

will be composed of

(a) Natural numbers (all of them are non-negative)
(b) Integer numbers (some of them are negative)
(c) Rational numbers (some of them are non-integers)
(d) Real numbers (some of them are irrational)
(e) Complex numbers (some of them are not real)

11. Let

$$\begin{aligned} p &= \log_3(2) \\ q &= \log_{\sqrt{3}}(4) \\ r &= \log_{1/3}\left(\sqrt{2}\right) \end{aligned}$$

Mark the correct option:

(a) $p < q < r$
(b) $r < q < p$
(c) $q < r < p$
(d) $p < r < q$
(e) $r < p < q$

12. By simplifying the expression

$$\frac{\cos^2(x) - \cot(x)}{\sin^2(x) - \tan(x)},$$

one gets

(a) $\sec^2(x)$
(b) $\sin^2(x)$
(c) $\tan^2(x)$
(d) $\cos^2(x)$
(e) $\cot^2(x)$

13. In order to

$$2 + \frac{4}{m} + \frac{8}{m^2} + \cdots = \frac{14}{2}$$

hold true, m should be

(a) 5
(b) 6
(c) 7
(d) 8
(e) none of the above values

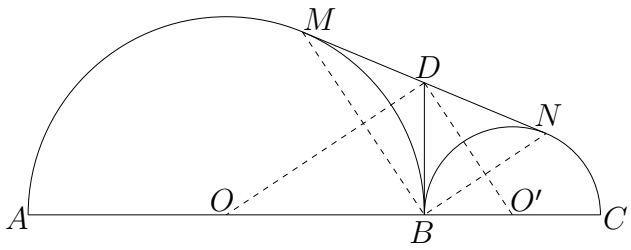

Figure 2: Tangent circles with tangent lines

14. The solution of the equation

$$\frac{(n+2)!(n-2)!}{(n+1)!(n-1)!} = 4$$

is a natural number that is also

(a) even

(b) a perfect cube

(c) larger than 10

(d) divisible by 5

(e) divisible by 3

15. A straight line passes through points $(3, 0)$ and $(0, 4)$. The distance from this line to the origin of the Cartesian plane is

(a) 2

(b) $\sqrt{5}$

(c) 12/5

(d) 5/2

(e) 3

16. In Figure 2, A, B and C are aligned points such that $\overline{AB} = 9$ is the diameter of the semicircle centered in O, while $\overline{BC} = 4$ is the diameter of the semicircle centered in O'. Both semicircles are tangent to each other and $\overline{BD}$ and $\overline{MN}$ are the common tangents to these circles. Mark the *wrong* option:

(a) $\overline{BD}$ is an altitude of the triangle ODO'

(b) $\overline{BD}$ is a median of the triangle MBN

(c) The triangle MBN is rectangle

(d) $\overline{MN} = 6.5$

(e) ODO' and MBN are similar triangles