2024-1																
Questão	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
Resposta	b	b	e	a	d	e	b	a	c	b	e	c	d	b	a	c

Instituto Tecnológico de Aeronáutica

Programa de Pós-Graduação em Engenharia de Infraestrutura Aeronáutica
Programa de Pós-Graduação em Engenharia Aeronáutica e Mecânica

Prova de Seleção – 1º semestre de 2024 – Questões de Matemática

13 de novembro de 2023

Nome do Candidato

Observações

1. Duração da prova: 90 minutos (uma hora e meia)
2. Não é permitido o uso de calculadoras nem softwares nem sites de cálculo numérico e/ou simbólico
3. Cada questão admite uma única resposta
4. Marque a alternativa que considerar correta no formulário Google enviado por e-mail

Questões em Inglês

1. Figure 1 shows a polyhedron composed of 6 equal isosceles trapezoids and 2 equal equilateral triangles. One says that two coloring schemes of a polyhedron are equal if a composition of any number of three-dimensional rotations (upside-down, left-right, etc., excluded any reflection) can make all sides with equal colors coincide. Suppose that three out of the six isosceles trapezoids should be painted in black. How many different coloring schemes can be performed in this condition?

   (a) 3 schemes.
   (b) 4 schemes.
   (c) 5 schemes.
   (d) 6 schemes.
   (e) 9 schemes.

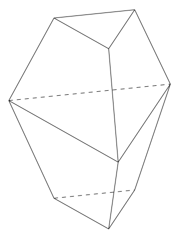

Figure 1: Polyhedron composed of isosceles trapezoids and equilateral triangles.

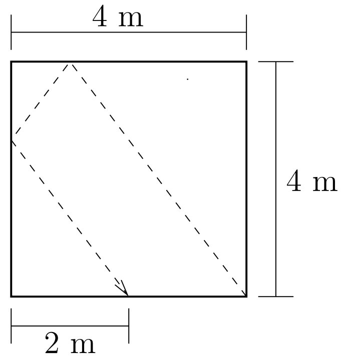

Figure 2: Light beam reflecting off mirror walls.

2. Figure 2 shows a square room with mirror walls 4 meters long. A beam of light enters from a corner, reflects off two walls and achieves the middle of a third wall. In the reflection, the angle of the incident light beam with respect to a wall is equal to the angle of the reflected light beam with respect to the same wall. The beam trajectory, which is depicted in the figure, has a total length of

   (a) $\sqrt{2} + \sqrt{5}$ meters.
   (b) 10 meters.
   (c) $5 + \sqrt{5} + 2\sqrt{2}$ meters.
   (d) $\sqrt{98}$ meters.
   (e) a value which is different from all previous options.

3. Let

$$A = \{1, 2, 4, 8, 16, 32\}$$

be a set of natural numbers. Take, for example, C, which is a subset of A:

$$C = \{1, 2, 4\}.$$

The sum of all elements of C is $\Sigma(C) = 7$. Mark the option which cannot be the sum of all elements of any subset of A:

   (a) 62
   (b) 51
   (c) 47
   (d) 23
   (e) All the numbers above can be obtained as the sum of the elements of a subset of A

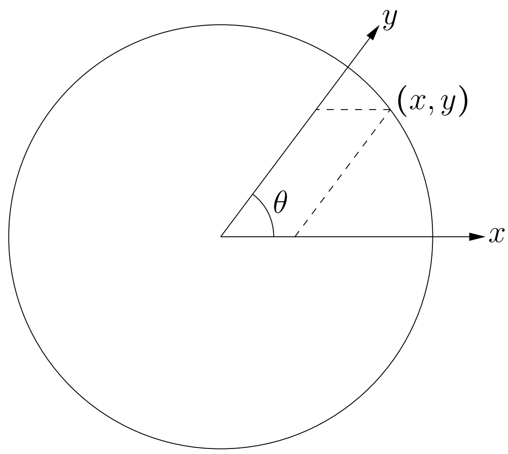

Figure 3: A circle and a non-orthogonal system of coordinates.

4. Figure 3 shows a circle of radius r that should be represented in a non-orthogonal system of coordinates (x, y). Axes x and y have the same unit length and the angle between these axes is θ, with $0 < \theta < \frac{\pi}{2}$. Under these conditions, the equation of the circle is

   (a) $x^2 + y^2 + 2xy\cos(\theta) = r^2$
   (b) $x^2 + y^2 - 2xy\cos(\theta) = r^2$
   (c) $x^2 + y^2 + 2xy\sin(\theta) = r^2$
   (d) $x^2 + y^2 - 2xy\sin(\theta) = r^2$
   (e) $x^2 + y^2 = r^2$

5. Figure 4 shows a map with a series of streets, where the horizontal streets are aligned with the west-east direction and the vertical streets are aligned with the north-south direction. All of them are one-way streets and the mandatory direction of traffic in the horizontal streets is eastbound, while the mandatory direction of traffic in the vertical streets is southbound. In the referred figure, one can see the corners A, B and C and an example of an allowed pathway from A to C that passes through B. How many different allowed trajectories can be made from A to C with the restriction of *not passing through the corner* B?

   (a) 16
   (b) 40
   (c) 56
   (d) 66
   (e) 116

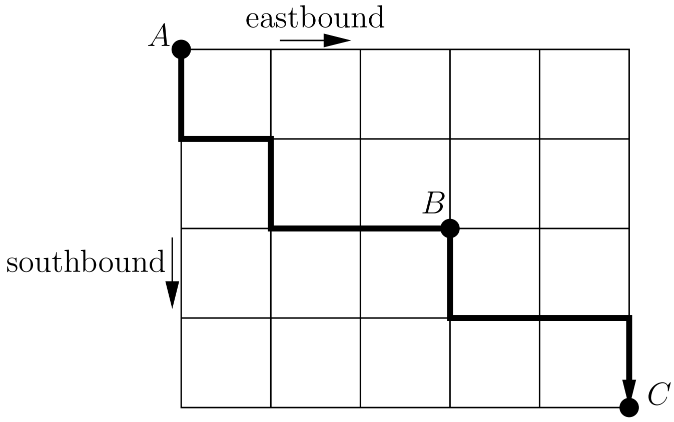

Figure 4: Map of streets aligned with the cardinal points.

6. The system of equations

$$\begin{cases} x^2 + y^2 & = & 25 \\ x \cdot y & = & 12 \end{cases}$$

has

(a) no real solutions
(b) only one real solution
(c) exactly two real solutions
(d) exactly three real solutions
(e) exactly four real solutions

7. The sum of the coefficients of all terms of second degree (x^2, y^2, x^2, xy, yz and xz) resulting from polynomial expansion of

$$(x - y + z - 1)^3$$

is

(a) -27
(b) -3
(c) 0
(d) 3
(e) 27

8. The equation $\dfrac{64}{32^x} = \dfrac{1}{8^{x^2}}$ has

(a) no real roots
(b) just one real root
(c) just two rational roots
(d) just three integer roots
(e) just four natural roots

9. A car runs over two different roads one after the other, at a constant but different speed in each of them. In the first road, it develops a speed of 40km/h and takes 3 hours. In the second one, it takes 2 hours. If the average speed is 48 km/h, the speed developed in the second road will be

(a) 54 km/h
(b) 56 km/h
(c) 60 km/h
(d) 63 km/h
(e) 64 km/h

10. A train runs with a constant ground speed V while two passengers run inside it in opposite directions. Passenger 1 runs in the same direction as the train with a speed $V_1 = 36$ km/h with respect to the train. Passenger 2 runs in the opposite direction with a speed $V_2 = 36$ km/h with respect to the train. The ground speed of passenger 1 is $V + V_1$, whereas the ground speed of passenger 2 is $V - V_2$. The ground speed of passenger 2 is 3/5 of the ground speed of passenger 1. The ground speed of passenger 2 is

(a) 72 km/h
(b) 108 km/h
(c) 144 km/h
(d) 180 km/h
(e) 216 km/h

11. The system of equations

$$\begin{cases} x + ay &= b \\ x + y &= a \end{cases}$$

has x and y as unknowns and a and b as parameters. Mark the *wrong* statement on the discussion of this system:

(a) if $a \neq 1$, the system has at least one solution
(b) if $a = b = 1$, the system solutions lie on a straight line in the plane (x, y)
(c) if $a = 1$ and $b = 2$, the system has no solution
(d) if $a = 1$ and $b = 3$, the system has no solution
(e) If $a + b = 1$, the system will always have a real solution

12. Figure 5 shows a quadrilateral together with *oriented* angles α and β between its sides. For $ABCD$ to be an inscribed quadrilateral and a circumscribed quadrilateral at the same time, it is necessary that

(a) $\alpha = 90°$ and $AB = CD$
(b) $\alpha = 60°$ and $AB = BC$
(c) $\alpha = 90°$ and $AB = AC$
(d) $\alpha = 60°$ and $AB = CD$
(e) $\alpha = 30°$ and $AD = DC$

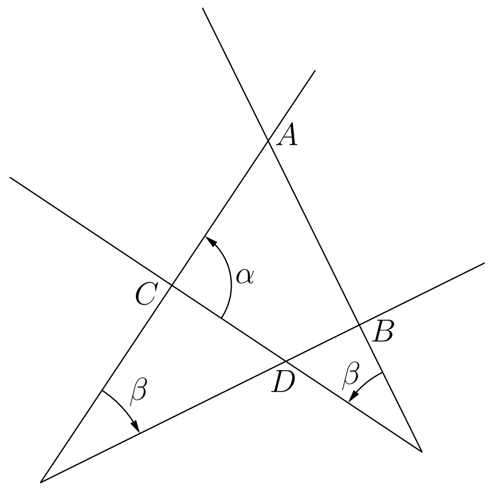

Figure 5: Quadrilateral with oriented angles between its sides.

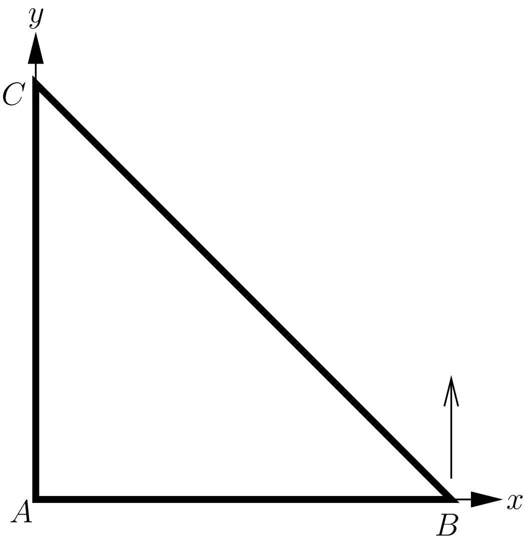

Figure 6: Triangle with a moving vertex.

13. Figure 6 shows a triangle ABC in which vertices $A(0,0)$ and $C(0,1)$ are fixed and vertex $B(1,t)$ moves in time t. The rate of change of the perimeter p of the triangle ABC for $t = 1$ is

   (a) $-\sqrt{2}/2$
   (b) 0
   (c) $\sqrt{2}/2 - \sqrt{5}/5$
   (d) $\sqrt{2}/2$
   (e) 1

14. Given $e = 2.7182818...$ as the basis of the natural logarithms, the value of the integral

$$\int_0^{\pi/2} e^{\sin^2(x)} \sin(x)\cos(x)\mathrm{d}x$$

is

(a) 0
(b) $\dfrac{e-1}{2}$
(c) 1
(d) $e-1$
(e) e^2

15. Which of the following options causes the equation $A^{12} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ to be false?

(a) $A = \begin{bmatrix} \sqrt{2}/2 & -\sqrt{2}/2 \\ \sqrt{2}/2 & \sqrt{2}/2 \end{bmatrix}$
(b) $A = \begin{bmatrix} \sqrt{3}/2 & -1/2 \\ 1/2 & \sqrt{3}/2 \end{bmatrix}$
(c) $A = \begin{bmatrix} 1/2 & -\sqrt{3}/2 \\ \sqrt{3}/2 & 1/2 \end{bmatrix}$
(d) $A = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$
(e) $A = \begin{bmatrix} 1/2 & \sqrt{3}/2 \\ -\sqrt{3}/2 & 1/2 \end{bmatrix}$

16. A plane contains five straight lines. There are no two of these lines which are parallel to each other. Moreover, no point of the plane is crossed by more than two of these lines. If these five lines divide the plane in n distinct regions, then n is equal to

(a) 14
(b) 15
(c) 16
(d) 17
(e) 32