



Instituto Tecnológico de Aeronáutica

Programa de Pós-Graduação em Engenharia de Infraestrutura Aeronáutica  
Programa de Pós-Graduação em Engenharia Aeronáutica e Mecânica

Prova de Seleção – 2º semestre de 2023 – Questões de Matemática

22 de maio de 2023

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Nome do Candidato

## Observações

1. Duração da prova: 90 minutos (uma hora e meia)
2. Não é permitido o uso de calculadoras nem softwares nem sites de cálculo numérico e/ou simbólico
3. Cada pergunta admite uma única resposta
4. Marque a alternativa que considerar correta no formulário Google enviado por e-mail

## Questões em Inglês

1. If  $[A]^2 = [A][A] = \begin{bmatrix} 4 & 10 \\ 0 & 9 \end{bmatrix}$ , one can say that
  - (a) there is only one matrix  $[A]$  that satisfies the equation above
  - (b) there are only two matrices  $[A]$  that satisfy the equation above
  - (c) there are only three matrices  $[A]$  that satisfy the equation above
  - (d) there are only four matrices  $[A]$  that satisfy the equation above
  - (e) there is no matrix  $[A]$  that satisfies the equation above

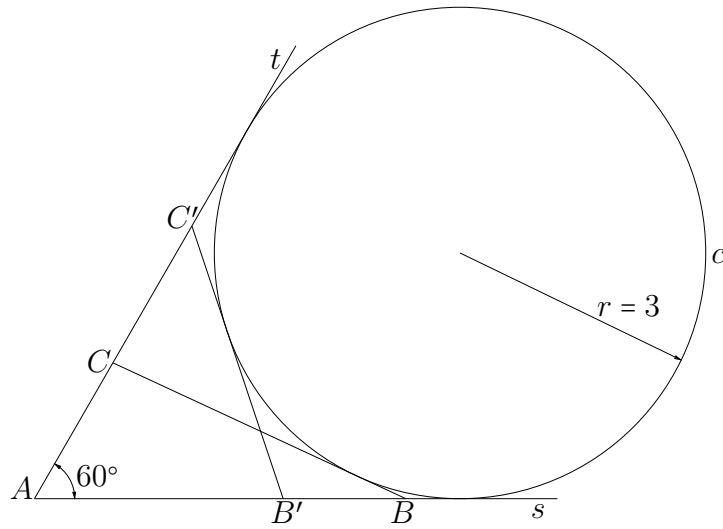


Figure 1: Triangles with the same escribed circle.

2. Figure 1 shows two triangles  $ABC$  and  $AB'C'$ . Both triangles have lines  $t$  and  $s$  supporting their sides; moreover, both triangles have the circle  $c$  as their *excircle* or *escribed circle*. Any other line tangent to  $c$  by the interior of  $t$  and  $s$  will produce a triangle with the same excircle  $c$ , with lines  $t$  and  $s$  supporting its sides. What is the value of the greatest possible area for all of these triangles?

- (a) 3
- (b)  $2\sqrt{2}$
- (c)  $2\sqrt{3}$
- (d) 4
- (e)  $3\sqrt{3}$

3. About the equations

$$x^3 - 6x - 9 = 0 \quad (1)$$

$$x^3 - 6x + 9 = 0 \quad (2)$$

one *cannot* say that

- (a) equation (1) has one natural root.
- (b) equation (2) has one integer root.
- (c) equation (1) has two complex roots.
- (d) equation (2) has two pure imaginary roots.
- (e) equation (2) has two complex roots.

4.  $p$  plus signs and  $q$  minus signs are to be placed in a row, such that  $p + q = n$  and  $p > q - 2$ . Considering that no two minus signs should be placed together, in how many different ways can these signs be placed?

- (a)  $C_q^{n-q+1}$
- (b)  $C_{p+q}^q$
- (c)  $\frac{(p+q)!}{q!}$
- (d)  $C_q^{p+1}$
- (e)  $(p+q-1)!$

Note:  $C_r^n = \frac{n!}{(n-r)!r!}$  is the  $r$ -combination of  $n$  distinct things

5. Three couples in a riverside are up to cross the river in a boat which can carry up to two persons. The three wives are jealous, such that no man should stay in any riverside or in the boat in the company of a wife of any other man without her husband. Within these constraints, what should be the minimal number of river crossings the boat should do in order to take the three couples to the other side?

- (a) 8
- (b) 9
- (c) 10
- (d) 11
- (e) 12

6. If  $S_n = \sum_{i=1}^n (-1)^i \sqrt{2^{i-1}} = -1 + \sqrt{2} - 2 + 2\sqrt{2} - 4 + 4\sqrt{2} - \dots + (-1)^n (\sqrt{2^{n-1}})$ , the value of  $\lim_{n \rightarrow \infty} S_n$  is:

- (a)  $-\infty$
- (b)  $-3 - \sqrt{2}$
- (c)  $\sqrt{2} + 1$
- (d)  $+\infty$
- (e) There is no defined limit for this series

7. Figure 2 shows a triangle and the nine concyclic points that define the *nine-point circle* (five of these points coincide in  $A$ ). Mark the correct coordinates of the center of the circle:

- (a)  $(4, 2.5)$
- (b)  $(4, 2\sqrt{2})$
- (c)  $(4.1, 2.4)$
- (d)  $(4.2, 2.5)$
- (e)  $(4.3, 2.4)$

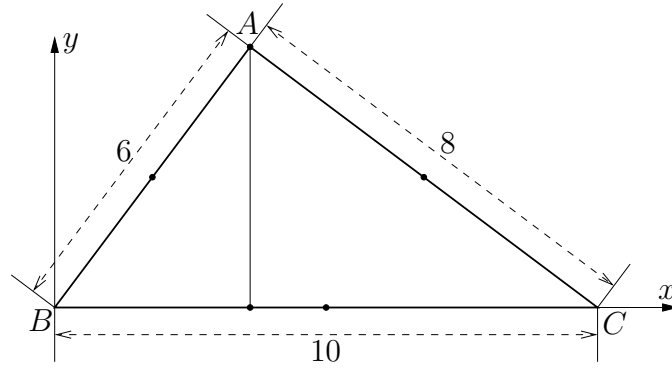


Figure 2: Triangle with the feet of its altitudes and the midpoints of its sides.

8. If  $n \in \mathbb{N}$ , mark the formula that *may not* result an integer number for all natural numbers  $n$ :

- (a)  $\frac{n^2 - n}{2}$
- (b)  $\frac{n^2 + n}{2}$
- (c)  $\frac{n^3 + 3n^2 + 2n}{6}$
- (d)  $\frac{n^3 - n}{6}$
- (e)  $\frac{n^3 + n}{6}$

9. Assuming  $e$  as the base of natural logarithm, the value of the integral

$$\int_0^{\infty} x e^{-x^2} dx$$

is

- (a)  $-1$
- (b)  $0$
- (c)  $\frac{1}{2}$
- (d)  $1$
- (e)  $\infty$

10. By simplifying  $\frac{n[n! + (n-1)!]}{(n+1)! - n!}$ , where  $n$  is a non-null natural number, one gets:

- (a)  $n$
- (b)  $\frac{n+1}{n}$
- (c)  $\frac{n-1}{n}$
- (d)  $\frac{n}{n+1}$
- (e)  $\frac{n}{n-1}$

11. About the system of equations

$$\begin{cases} x - y - z &= 3 \\ -x + y - z &= 4 \\ x - y + z &= 5 \end{cases}$$

one can say that

- (a) this sytem has no real solutions.
  - (b) this sytem has one real solution.
  - (c) this sytem has two real solutions.
  - (d) this sytem has more than two solutions.
  - (e) more information is needed in order to discuss this system.
12. Let  $f(x) = \sqrt{1-x^2}$ . For what real value of  $x$  one gets a real value for  $f'(x) = f(x)$ ?

(a)  $\frac{1-\sqrt{5}}{2}$ .

(b) 0.

(c)  $\frac{\sqrt{2}}{2}$ .

(d)  $\arcsin\left(\frac{1}{2}\right)$ .

(e) There is no real value of  $x$  that holds  $f'(x) = f(x)$  true.

13. The manager of a clothing store used to sell a shirt by R\$ 150,00. In the beginning of a certain month, he raised the price of that item by 20%. After two weeks the sales decline and the manager decides to reduce price of this item by 20%. Then, the new the price of that shirt will be

(a) R\$ 120,00.

(b) R\$ 139,00.

(c) R\$ 150,00.

(d) R\$ 144,00.

(e) none of the previous options.

14. The real root of equation

$$\left(7^x + 2\sqrt{10}\right)\left(7^x - 2\sqrt{10}\right) = 9$$

is

- (a) a negative irrational number.
- (b) a positive irrational number.
- (c) a positive even number.
- (d) a negative integer number.
- (e) a positive odd number.

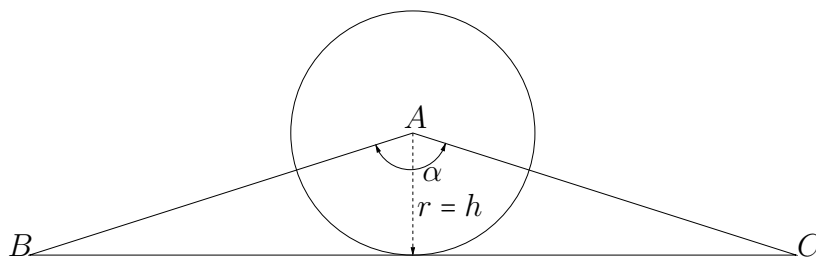


Figure 3: Triangle and circle with same area.

15. Assuming  $e$  as the base of natural logarithm, the value of the integral

$$\int_0^{e-1} \frac{x+2}{x+1} dx$$

is

- (a)  $e - 1$
  - (b)  $2$
  - (c)  $e$
  - (d)  $e + 1$
  - (e)  $\infty$
16. Figure 3 shows a circle and an isosceles triangle which have the same area. About this figure, three statements are posed:
- $BC = 2\pi r$
  - $\widehat{BAC} = 2 \arctan(\pi)$
  - $AC = r\sqrt{\pi^2 - 1}$

Mark the correct choice about the discussion of these statements:

- (a) All three statements are false.
- (b) There are two false statements.
- (c) There is only one false statement.
- (d) All statements are true.
- (e) These statements cannot be discussed if  $r = h$  is not given.