Instituto Tecnológico de Aeronáutica

Programa de Pós-Graduação em Engenharia de Infraestrutura Aeronáutica
Programa de Pós-Graduação em Engenharia Aeronáutica e Mecânica

Prova de Seleção – 1º semestre de 2023 – Questões de Matemática

21 de novembro de 2022

Nome do Candidato

Observações

1. Duração da prova: 90 minutos (uma hora e meia)
2. Não é permitido o uso de calculadoras nem softwares nem sites de cálculo numérico e/ou simbólico
3. Cada pergunta admite uma única resposta
4. Marque a alternativa que considerar correta no formulário Google enviado por e-mail

Questões em Inglês

1. Figure 1 shows a Penrose tiling, which is a covering of the plane with non-overlapping figures, composing a non-periodic pattern. In this figure, the Penrose tiling is composed of two different kinds of rhombi, one painted in gray and the other painted in white. About the geometry of this tiling, mark the *wrong* statement:

   (a) For the gray rhombi, the ratio of the smaller diagonal length to the side length is the golden ratio

   (b) All the internal angles of the two kinds of rhombi are in arithmetic progression

   (c) The ratio of the area of the gray rhombus to the area of the white rhombus is the golden ratio

   (d) For the gray rhombi, the smaller internal angle is equal to the internal angle of a stellated regular pentagon

   (e) For the gray rhombi, the larger internal angle is equal to the internal angle of a non-stellated regular pentagon

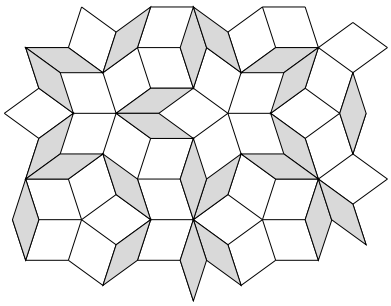

Figure 1: Penrose tiling

Hint: $\sin\left(\frac{\pi}{10}\right) = \frac{\sqrt{5}-1}{4}$

2. In how many ways can four men and four women sit around a circular table if no man can sit beside another one?

   (a) 36
   (b) 54
   (c) 72
   (d) 108
   (e) 144

3. About the equation

$$\sqrt{3}x^2 + \frac{\sqrt{3}}{\sqrt{2}+1}x - \sqrt{6} = 0,$$

   one *cannot* say that

   (a) It has one integer solution
   (b) It has one negative solution
   (c) It has one positive solution
   (d) It has one irrational solution
   (e) It has no real solutions

4. In order for the equation

$$\begin{Bmatrix} x & y & z & 1 \end{Bmatrix} \begin{bmatrix} a & e & f & g \\ e & b & h & i \\ f & h & c & j \\ g & i & j & d \end{bmatrix} \begin{Bmatrix} x \\ y \\ z \\ 1 \end{Bmatrix} = 0$$

   to describe a *non-degenerated* sphere in $\mathbb{R}^3$, mark the *wrong* constraint for the matrix coefficients:

   (a) $e = 0$
   (b) $f = 0$
   (c) $h = 0$
   (d) $a = b = c$
   (e) $a \cdot d > 0$

5. The coefficient of x^{17} in the development of $(1 + x^5 + x^7)^{20}$ is

   (a) 0
   (b) 1210
   (c) 1710
   (d) 3420
   (e) 4000

6. When Andy and Robert worked together, they swept a 280 m^2 floor in four hours. In another day, Robert worked alone and took seven hours to do the same task. If Andy will sweep alone an area of 240 m^2, what is the expected time for him to do it?

   (a) 6 hours
   (b) 6.5 hours
   (c) 7 hours
   (d) 7.5 hours
   (e) 8 hours

7. About the function

$$f(x) = \cosh(x)\cos(x),$$

   mark the *wrong* statement (otherwise, mark the last option)

   (a) $\dfrac{\mathrm{d}}{\mathrm{d}x} f(x) = \sinh(x)\cos(x) - \cosh(x)\sin(x)$
   (b) $\dfrac{\mathrm{d}^2}{\mathrm{d}x^2}\left[f\left(\dfrac{x}{\sqrt{2}}\right)\right] = -\sinh\left(\dfrac{x}{\sqrt{2}}\right)\sin\left(\dfrac{x}{\sqrt{2}}\right)$
   (c) $\dfrac{\mathrm{d}^3}{\mathrm{d}x^3} f(x) = -2\left[\cosh(x)\sin(x) + \sinh(x)\cos(x)\right]$
   (d) $\dfrac{\mathrm{d}^4}{\mathrm{d}x^4}\left[f\left(\dfrac{x}{\sqrt{2}}\right)\right] = -f\left(\dfrac{x}{\sqrt{2}}\right)$
   (e) All statements are true

8. The solution of the system of equations

$$\begin{cases} x + 3y + 3z + t &= 1 \\ x - 3y + 3z - t &= 27 \\ x + 2y &= 0 \\ \dfrac{t}{y} &= \dfrac{1}{4} \end{cases}$$

   is

   (a) $x = 4$, $y = 3$, $z = 2$ and $t = 1$
   (b) $x = 8$, $y = 4$, $z = 2$ and $t = 1$
   (c) $x = 8$, $y = -4$, $z = 2$ and $t = -1$
   (d) $x = 3$, $y = 3$, $z = 3$ and $t = 3$
   (e) This system admits more than one solution

9. Which choice is equal to $\log_a\left(\sqrt{\frac{a^3}{b}}\right)$ for all $a, b > 0$?

   (a) $\frac{1}{2}[\ln(3) - \log_a(b)]$

   (b) $\sqrt{\ln(3) - \log_a(b)}$

   (c) $\frac{1}{2}\left[3 - \frac{1}{\log_b(a)}\right]$

   (d) $\sqrt{3 + \log_b(a)}$

   (e) None of the above is valid for all $a, b > 0$

10. About the equation

$$x^4 + 10x^3 + 25x^2 + y^2 - 10y + 25 = 0,$$

    one can say that

   (a) this equation has no complex solution

   (b) this equation has two distinct real solutions

   (c) this equation has one real solution with an irrational value of x or y

   (d) this equation has only one solution with integer values for x and y

   (e) this equation has no real solution (x, y) with $x < y$

11. The curve $y = \arccos(x)$ defines a function in the region $(x, y) \in [-1, 1] \times [0, \pi]$. Mark the option that defines the line that is tangent to this function in $y = \frac{\pi}{3}$:

   (a) $y = \frac{3 + \sqrt{3}\pi}{6} - \frac{\sqrt{3}}{2}x$

   (b) $y = \frac{2\pi}{3} - \frac{2\sqrt{3}}{3}x$

   (c) $y = \frac{\pi + \sqrt{3}}{3} - \frac{2\sqrt{3}}{3}x$

   (d) $\frac{3}{2} - \frac{\pi}{3}x$

   (e) There are several solutions for this problem

12. Figure 2 depicts circle c_1, which remains fixed while circle c_2 moves, rolling over c_1 without slipping. Circle c_2 starts at the top of c_1 and ends its movement by the right side of c_1, so that its center describes a quarter turn in relation to the center of c_1. Mark the correct value of the angle of rotation of c_2 over its own center, given that c_1 radius is three times the radius of c_2:

   (a) $\pi/2$

   (b) π

   (c) $3\pi/2$

   (d) $4\pi/3$

   (e) 2π

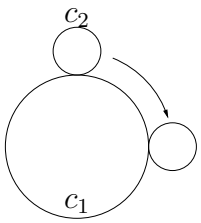

Figure 2: Circle

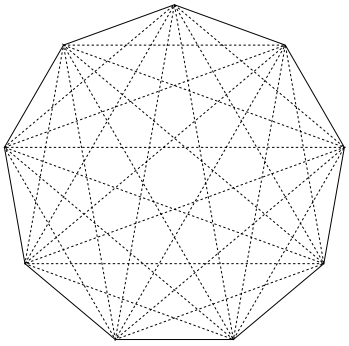

Figure 3: Regular nonagon with diagonals

13. Four points in space are given, not all in the same plane. How many planes can be drawn which are equidistant from these points?

    (a) 6 planes

    (b) 7 planes

    (c) 8 planes

    (d) 10 planes

    (e) 12 planes

14. Figure 3 shows a regular nonagon or enneagon, with its internal diagonals depicted as dashed lines. The number of points of intersection between these internal diagonals is

    (a) 90

    (b) 108

    (c) 126

    (d) 144

    (e) 162

15. About the sum

$$S = 1 - \frac{1+i}{2} + \left(\frac{1+i}{2}\right)^2 - \left(\frac{1+i}{2}\right)^3 + \cdots = \sum_{k=0}^{\infty} (-1)^k \left(\frac{1+i}{2}\right)^k,$$

in which i is the imaginary unit, one can say that

(a) $S = \frac{3}{4} - \frac{1}{4}i$

(b) $S = 0.6 - 0.2i$

(c) $S = \infty$, i.e., the sum diverges

(d) The sum oscillates indefinitely; hence, it does not have a limit

(e) This sum cannot be determined, as the formula for an infinite geometric progression is not defined for complex numbers

16. If $\frac{1}{x} = 2\frac{2}{5}$ then $\left(\frac{1}{x+1}\right)^2 =$

(a) $\frac{25}{64}$

(b) $\frac{5}{8}$

(c) $\frac{144}{289}$

(d) $\frac{289}{144}$

(e) $\frac{64}{25}$

Gabarito Matemática:

1	e (anulada por múltiplas respostas)
2	e
3	e
4	e
5	d
6	e
7	e
8	c
9	e
10	b
11	c
12	e
13	b
14	c
15	b
16	c