Instituto Tecnológico de Aeronáutica

Programa de Pós-Graduação em Engenharia de Infraestrutura Aeronáutica
Programa de Pós-Graduação em Engenharia Aeronáutica e Mecânica

Prova de Seleção – 2º semestre de 2022 – Questões de Matemática

16 de maio de 2022

_____
Nome do Candidato

Observações

1. Duração da prova: 90 minutos (uma hora e meia)

2. Não é permitido o uso de calculadoras nem softwares nem sites de cálculo numérico e/ou simbólico

3. Cada pergunta admite uma única resposta

4. Marque a alternativa que considerar correta no formulário Google enviado por e-mail

Questões em Inglês

1. The coefficient of x^4 obtained after expanding and summing up $(1 + x)(1 - x)^n$ is

   (a) $\dfrac{(n-1)(n-2)(n-3)(n-4)}{12}$

   (b) $\dfrac{n(n-1)(n-2)(n-4)}{24}$

   (c) $\dfrac{n(n-1)(n-2)(n-7)}{24}$

   (d) $\dfrac{n(n-1)(n-2)(n-5)}{24}$
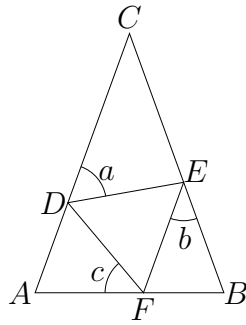
   (e) $\dfrac{n(n-1)(n-2)(n-5)}{120}$

Figure 1: Equilateral triangle inscribed in an isosceles triangle

2. In Figure 1 CA and CB are the equal sides of an isosceles triangle ABC, in which an equilateral triangle DEF is inscribed. Designate angle CDE by a, FEB by b and AFD by c. Then:

(a) $a = \dfrac{c+b}{2}$

(b) $a = \dfrac{c-b}{2}$

(c) $c = \dfrac{a-b}{2}$

(d) $c = \dfrac{a-b}{b}$

(e) none of these

3. Two reservoirs A and B are simultaneously drained by two pumps that ensure constant drain rates of $Q_A = 3$ L/s and $Q_B = 2$ L/s, respectively, until they get depleted. Both reservoirs A and B discharge into reservoir C, which is also drained by a third pump at a constant rate of $Q_C = 4$ L/s until it gets depleted. At an initial instant $t = 0$, reservoirs A, B and C contain 180 L, 100 L and 200 L of water, respectively. Reservoir C has a capacity of 250 L and if it is flooded, the excess of water is simply dumped. Mark the *wrong* statement about this pumping process:

(a) C will be full in $t = 1$ minute 40 seconds

(b) C will be empty in $t = 2$ minutes

(c) C will not flood and dump in any time

(d) A will be empty in $t = 60$ seconds

(e) B will be empty in $t = 50$ seconds

4. Which of the following equations has a root in common with $x^2 - 5x + 4 = 0$?

(a) $x^2 + 1 = 0$

(b) $x^2 - x - 2 = 0$

(c) $x^2 - 10x - 5 = 0$

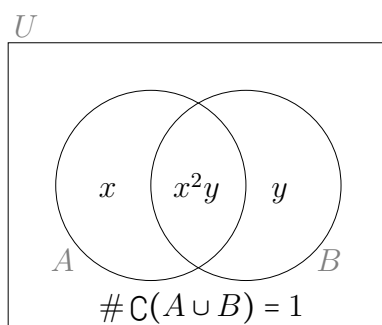(d) $3x^2 - 3 = 0$

(e) $x^2 - 2x - 3 = 0$

Figure 2: Euler diagram with the number of elements in each subset (# is the number of elements of a set and $\complement$ is the complement set in relation to the universe U)

5. About the system of equations

$$\begin{cases} x - y^2 + z^3 & = & -2 \\ x + y^2 - z^3 & = & 8 \\ x - y^2 - z^3 & = & 0 \end{cases}$$

one can say that

(a) it has 6 distinct sets of real roots for x, y and z

(b) it has 3 distinct sets of real roots for x, y and z

(c) it has 2 distinct sets of real roots for x, y and z

(d) it has only 1 set of real roots for x, y and z

(e) it has no set of real roots for x, y and z

6. Let A and B be two *statistically independent* events. Figure 2 shows the Euler diagram with the number of elements in each subset described as a function of two variables x and y, with $x \cdot y \neq 0$. Given the information provided, mark the correct statement:

(a) Both x and y can be determined with the information given

(b) Variable x can be determined with the information given while y cannot be determined

(c) Variable y can be determined with the information given while x cannot be determined

(d) Neither x nor y can be determined with the information given, but a relation between them can be stated

(e) Neither x nor y can be determined with the information given, and no relation between them can be stated

7. What matrix has its determinant equal to 1?

(a) $\begin{bmatrix} \dfrac{\sqrt{2}}{2} & \dfrac{\sqrt{2}}{2} \\ \dfrac{\sqrt{2}}{2} & -\dfrac{\sqrt{2}}{2} \end{bmatrix}$

(b) $\begin{bmatrix} 0 & \dfrac{\sqrt{2}}{2} - \dfrac{\sqrt{2}}{2}i \\ \dfrac{\sqrt{2}}{2} + \dfrac{\sqrt{2}}{2}i & 0 \end{bmatrix}$

(c) $\begin{bmatrix} \dfrac{\sqrt{2}}{2}i & -\dfrac{\sqrt{2}}{4} - \dfrac{\sqrt{2}}{4}i \\ \dfrac{\sqrt{2}}{4} + \dfrac{\sqrt{2}}{4}i & -\dfrac{\sqrt{2}}{2}i \end{bmatrix}$

(d) $\begin{bmatrix} 1 & 1+i \\ 1-i & -1 \end{bmatrix}$

(e) None of them has determinant equal to 1.

3

8. Table 1 shows the car rental charges at Thrifty Agency. The daily rate applies for each day or fraction of a day in excess of any multiple of a 7-day week, up to the charge per week. Maria rented a car of one of the types indicated in the table; about this rental, two statements are posed:

   I Maria's total rental charge, based only on the rates specified, was $92.

   II Maria rented the car for 10 days.

In order to know which type of car Maria rented, what could be said about these satements?

   (a) Statement I *alone* is sufficient, but statement II alone is not sufficient to answer the question asked;

   (b) Statement II *alone* is sufficient, but statement I alone is not sufficient to answer the question asked;

   (c) *Both* statements I and II *together* are sufficient to answer the question asked, but *neither* statement *alone* is sufficient;

   (d) *Each* statement *alone* is sufficient to answer the question asked;

   (e) Statements I and II *together* are *not* sufficient to answer the question asked, and additional data specific to the problem are needed.

9. Let

$$
\begin{aligned}
f(x) &= \arcsin(x), \forall x \in [-1, 1] \\
g(x) &= \arccos(x), \forall x \in [-1, 1] \\
h(x) &= \arctan(x), \forall x \in [-\infty, \infty]
\end{aligned}
$$

Mark the *wrong* statement about them (otherwise, mark the last one):

   (a) $f'(x) + g'(x) = 0$

   (b) $f^{-1}(g(x)) = g^{-1}(f(x))$

   (c) $[f'(x)]^{-1}(g'(x)) = -1$

   (d) $\dfrac{1}{h'(x)} - \dfrac{1}{f'(x) \cdot g'(x)} = 2$

   (e) All statements are correct.

Table 1: Car rental charges at Thrifty Agency

Car Type	Charge per day	Charge per Week (7 days)
Economy	$14	$50
Compact	$15	$60
Midsize	$16	$70
Standard	$17	$80
Luxury	$19.50	$100

10. Five bottles of wine X cost a total of $64.00 and 7 bottles of wine Y cost a total of $163.80. At the same uniform rates per bottles, what is the total cost of 3 bottles of wine X and 2 bottles of wine Y ?

   (a) $36.2

   (b) $85.2

   (c) $94.9

   (d) $95.8

   (e) $98.8

11. The equation

$$-\log_{1/\sqrt{e}}(x) \ = \ \ln(4x-3)$$

   (a) has no real roots

   (b) has one positive real root

   (c) has one negative real root

   (d) has two distinct real roots

   (e) has more than two positive real roots

12. Working alone, Alice can complete a certain kind of job in 18 hours. Alice and Bob, working together at their respective rates, can complete the same kind of job in 12 hours. In how many hours can Bob, working alone, complete this kind of job?

   (a) 36

   (b) 24

   (c) 18

   (d) 12

   (e) 6

13. Mark the *wrong* statement:

   (a) $\operatorname{sen}(15°) = \dfrac{\sqrt{2}}{4}\left(\sqrt{3}-1\right)$

   (b) $\cos(15°) = \dfrac{\sqrt{2}}{4}\left(\sqrt{3}+1\right)$

   (c) $\tan(15°) = 2-\sqrt{3}$

   (d) $\cos(105°) = \dfrac{\sqrt{2}}{4}\left(1-\sqrt{3}\right)$

   (e) $\tan(105°) = -\left(2-\sqrt{3}\right)$

14. One hour after Yasmim started walking from A to B, a distance of 22.5 kilometers, Julia started walking along the same road from B to A. If Yasmim's walking rate was 1.5 kilometers per hour and Julia's was 2 kilometers per hour, how many kilometers had Julia walked when they met?

    (a) 12

    (b) 11.5

    (c) 11

    (d) 10.5

    (e) 9.75

15. About the sequence

$$a_n = \cos^{n+1}(n\,\theta),$$

    mark the right statement (if all statements are true, mark the last one):

    (a) it is convergent for any value of $\theta \in (0, \pi)$

    (b) it is convergent for any value of $\theta \in (\pi, 2\pi)$

    (c) it is convergent for $\theta = \pi$

    (d) it is convergent for $\theta = 2\pi$

    (e) all the previous statements are true

16. A rectangular floor is covered by a rug except for a strip c meters wide along each of the four edges. If the floor is a meters by b meters, what is the area of the rug, in square meters?

    (a) $ab - c(a+b)$

    (b) $ab - 2c(a+b)$

    (c) $ab - c^2$

    (d) $(a-c)(b-c)$

    (e) $(a-2c)(b-2c)$

gabarito:

1) c
2) e
3) a
4) d
5) c
6) b
7) e
8) a
9) e
10) b
11) d
12) a
13) e
14) a
15) e Errado, respostas corretas: c & d
16) e