Instituto Tecnológico de Aeronáutica

Programa de Pós-Graduação em Engenharia de Infraestrutura Aeronáutica
Programa de Pós-Graduação em Engenharia Aeronáutica e Mecânica

Prova de Seleção – 1º semestre de 2022 – Questões de Matemática

8 de novembro de 2021

Nome do Candidato

Observações

1. Duração da prova: 90 minutos (uma hora e meia)
2. Não é permitido o uso de calculadoras ou outros dispositivos eletrônicos
3. Cada pergunta admite uma única resposta
4. Marque a alternativa que considerar correta no formulário Google enviado por e-mail

Questões em Inglês

1. Two sequences of numbers a_i and b_i are defined by the recurrence relation

$$a_{i+1} = \frac{4}{5}a_i - \frac{3}{5}b_i \quad (1)$$

$$b_{i+1} = \frac{3}{5}a_i + \frac{4}{5}b_i \quad (2)$$

and by the initial values $a_0 = 1$ and $b_0 = 0$. Mark the *wrong* option:

(a) a_i and b_i are bounded for any $i \in \mathbb{N}$

(b) b_i is negative for some values of $i \in \mathbb{N}$

(c) $a_i = 1$ and $b_i = 0$ for some $i > 0 \in \mathbb{N}$

(d) $a_{i+2} = \frac{7}{25}a_i - \frac{24}{25}b_i$ for any $i \in \mathbb{N}$

(e) $a_i = \frac{4}{5}a_{i+1} + \frac{3}{5}b_{i+1}$ for any $i \in \mathbb{N}$

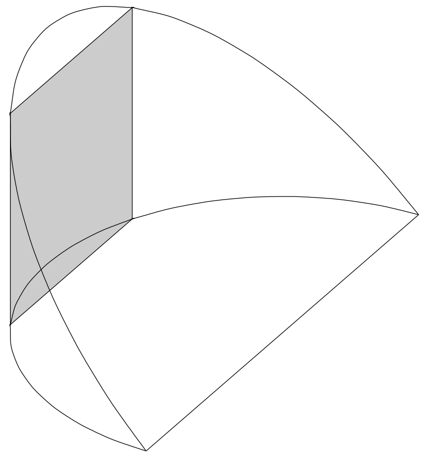

Figure 1: Half of a circular cylinder cut by a plane (not in scale)

A	60	...
B	...	50%
C	45	...
D	...	15%
Σ	...	100%

Table 1: Porcentages and absolute quantities

2. Figure 1 shows a half right circular cylinder of radius r, which is cut by a plane tilted by $45°$ in relation to the cylinder basis. A plane perpendicular to the cylinder basis is drawn and its intersection to cylinder basis is paralel to the intersecion of this basis and the tilted plane. The intersection of this plane and the cut cylinder is the rectangle shown in the cited Figure coloured in gray. If the rectangle has the maximal area A in these conditions, one could say that

   (a) $A = \dfrac{r^2}{2}$

   (b) $A = r^2$

   (c) $A = r^2 \cos(1)\mathrm{sen}(1)$, with trigonometric functions arguments expressed in radians

   (d) $A = 2r^2$

   (e) None of the above

3. In order to complete Table 1 of four numbers A, B, C and D in relation to their sum, mark the *wrong* statement:

   (a) Their sum is 300

   (b) A corresponds to 20% of the sum

   (c) $B = 150$

   (d) C corresponds to 25% of the sum

   (e) B and D sum up 195

4. Two statements are given about three numbers x, y and z:

   I. $x + \dfrac{1}{y} = 3$

   II. $\dfrac{y}{xy+1} + \dfrac{1}{z} = 5$

   If it is asked to determine the value of z, one can say that

   (a) statement I *alone* is sufficient, but statement II *alone* is *not* sufficient to answer the question asked;
   (b) statement II *alone* is sufficient, but statement I *alone* is *not* sufficient to answer the question asked;
   (c) *both* statements I and II *together* are sufficient to answer the question asked, but *neither* statement *alone* is sufficient;
   (d) *each* statement *alone* is sufficient to answer the question asked;
   (e) statements I and II *together* are *not* sufficient to answer the question asked, and additional data specific to the problem are needed.

5. What is the correct formula to calculate $\sec(a+b)$?

   (a) $\dfrac{1}{\tan(a)+\tan(b)}$
   (b) $\sec(a)+\sec(b)$
   (c) $\dfrac{\sec(a)+\sec(b)}{1-\sec(b)+\sec(a)}$
   (d) $\dfrac{\sec(a)\sec(b)}{1-\tan(a)\tan(b)}$
   (e) $\sec(a)\csc(b)+\sec(b)\csc(a)$

6. About the roots of the polynomial equation $x^6 - 64 = 0$, one can say that

   (a) it has 6 non rational roots
   (b) it has 4 non integer roots
   (c) it has 2 integer roots
   (d) it has 6 natural roots
   (e) it is not possible to discuss the roots of this polynomial

7. Among the options below, which one produces a rational number when divided by $\ln(x+1)$ for all $x > 1$?

   (a) $\ln(x^4 - 4x^3 + 6x^2 - 4x + 1)$
   (b) $\ln(x^2 - 1)$
   (c) $\ln(x^2 - 2x + 1)$
   (d) $\ln\left(\sqrt{x-1}\right)$
   (e) $\ln(x^3 - 1) - \ln(x^2 - x + 1)$

8. If $0 \le x < y \le 9$, how many different integer values the ordered pair (x, y) can assume?
   (a) 100
   (b) 72
   (c) 66
   (d) 55
   (e) 45

9. If $a, b \in \left[0, \frac{\pi}{2}\right)$, in order to the following equation be true,

$$\tan(a+b) = [\tan(a) + \tan(b)]\left\{1 + \tan(a)\tan(b) + [\tan(a)\tan(b)]^2 + \cdots\right\}$$

   what statement describe the correct additional restrictions for a and b?
   (a) $a - b < \frac{\pi}{2}$
   (b) $a + b < \frac{\pi}{2}$
   (c) $a = b$
   (d) $a \ge b$
   (e) This equation is analytic and is valid within the intervals specified without additional restrictions

10. The number of real solutions of the equation $\cos(2\theta) + 3\text{sen}(\theta) - 2 = 0$ in the interval $[0, \pi]$ is
   (a) 0
   (b) 1
   (c) 2
   (d) 3
   (e) 4

11. Let $x = a$ and $y = b$ be the *real* solutions for the following system of equations:

$$\begin{cases} \log(x) - \log(y) = 1 \\ x^2 - 198y = 4. \end{cases}$$

   So, the value of $\frac{a}{5} + b$ is
   (a) 3 or -3
   (b) 6
   (c) 6 or -0.06
   (d) 9
   (e) 9 ou -9

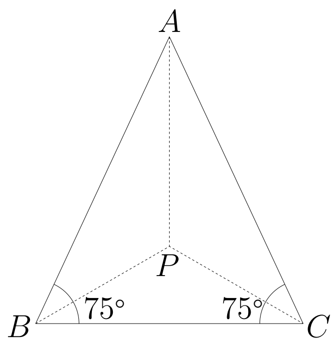

Figure 2: Isosceles triangle (drawing not in scale)

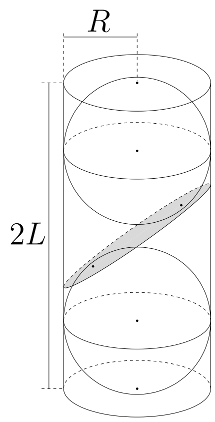

Figure 3: Cylinder and a plane tangent to two spheres

12. Let ABC be an isosceles triangle which has BC as its base and the sides AB and AC are opposite to $75°$ angles, as shown in Figure 2. Let P be a point inside the triangle such that the sum of the distances $\overline{PA} + \overline{PB} + \overline{PC}$ has minimum value. One can say that

    (a) P is the circumcenter of ABC
    (b) P is the incenter of ABC
    (c) P is in the midpoint of BC
    (d) P is over the line segment between the incenter and the orthocenter of ABC
    (e) P is over the line segment between the barycenter and the incenter of ABC

13. Figure 3 shows a cylinder with height $2L$ and radius R, such that $L > 2R$, and there are two spheres with radius R inside the cylinder, tangent to the top and the bottom of the cylinder. A plane which intersect the cylinder and is tangent to the two spheres will define a conic section with area

    (a) $\pi R\,(L - R)$
    (b) $\pi R\,(L + R)$
    (c) $\pi R\dfrac{L + R}{2}$
    (d) $\pi R\,L$
    (e) $\pi R\dfrac{L - R}{2}$

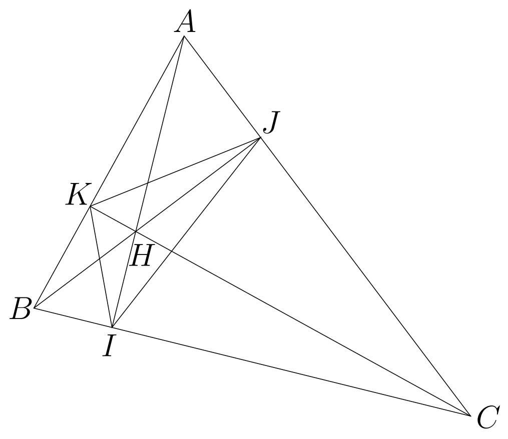

Figure 4: Triangle with orthocenter and altitude feets

14. Ten coins are stacked, some with heads up, other with tails up. How many ways of stacking these coins are there, in order to exactly six coins be with heads up?
   (a) 176
   (b) 210
   (c) 386
   (d) 596
   (e) 638

15. Figure 4 shows a triangle ABC, where H is its orthocenter and I, J and K are the feet of the three altitudes from A, B and C, respectively. Mark the *wrong* statement about this figure:
   (a) Quadrilaterals $AJHK$, $BKHI$ and $CIHJ$ can be inscribed in circles
   (b) AI, BJ and CK are bissectors of the angles $\widehat{JIK}$, $\widehat{KJI}$ and $\widehat{IKJ}$, respectively
   (c) $\widehat{KAH} = \widehat{KJH}$
   (d) H is the circumcenter of triangle IJK
   (e) Among all the triangles with vertices over the sides of triangle ABC, IJK is the triangle of smallest perimeter

16. I have three times the age you had by the time I had the double of the age that you have now. When you reach the age I have now, our ages will sum 69 years. Hence, I am
   (a) 24 years old
   (b) 27 years old
   (c) 32 years old
   (d) 35 years old
   (e) 42 years old