Instituto Tecnológico de Aeronáutica

Programa de Pós-Graduação em Engenharia de Infraestrutura Aeronáutica
Programa de Pós-Graduação em Engenharia Aeronáutica e Mecânica

Prova de Seleção – 2º semestre de 2021 – Questões de Matemática

24 de maio de 2021

Nome do Candidato

Observações

1. Duração da prova: 90 minutos (uma hora e meia)
2. Não é permitido o uso de calculadoras ou outros dispositivos eletrônicos
3. Cada pergunta admite uma única resposta
4. Marque a alternativa que considerar correta no formulário Google enviado por e-mail

Questões em Inglês

1. In a small car factory, a team of workers assembles the parts while other team paints the car assembled. The two tasks are sequential and, although all the six workers can perform both tasks, each worker should be designated to just one of the tasks. One worker alone takes four weeks to assemble the parts of one car while one worker alone takes one week to paint one car. Both tasks are fully scalable, in the sense that two employees working together should take two weeks to assemble one car and two employees working together should take half a week to paint one car. Which designation should be used in the factory in order to produce each car in the shortest possible time?

   (a) One worker at the assemblage and five workers in the painwork
   (b) Two workers at the assemblage and four workers in the painwork
   (c) Three workers at the assemblage and three workers in the painwork
   (d) Four workers at the assemblage and two workers in the painwork
   (e) Five workers at the assemblage and one worker in the painwork

2. In the system of linear equations

$$\begin{cases} x_1 = 1 \\ x_1 + x_2 = 3 \\ x_1 + x_2 + x_3 = 7 \\ \vdots \\ x_1 + x_2 + \cdots + x_n = 2^n - 1 \end{cases}$$

x_{10} =?

(a) 255
(b) 333
(c) 512
(d) 1024
(e) none of the above

3. The equation

$$\ln(x) + \ln(4 - x) = 1$$

(a) has integer roots
(b) has non-integer rational roots
(c) has non-rational real roots
(d) has no real roots
(e) none of the above

4. Two statements are given about three numbers x, y and z:

I. $\frac{1}{x} + \frac{1}{y} + \frac{1}{z} = 3$

II. $\frac{1}{xy} + \frac{1}{yz} + \frac{1}{xz} = 5$

If it is asked to determine the value of $\frac{x + y + z}{xy + yz + xz}$, one can say that

(a) statement I *alone* is sufficient, but statement II *alone* is *not* sufficient to answer the question asked;
(b) statement II *alone* is sufficient, but statement I *alone* is *not* sufficient to answer the question asked;
(c) *both* statements I and II *together* are sufficient to answer the question asked, but *neither* statement *alone* is sufficient;
(d) *each* statement *alone* is sufficient to answer the question asked;
(e) statements I and II *together* are *not* sufficient to answer the question asked, and additional data specific to the problem are needed.

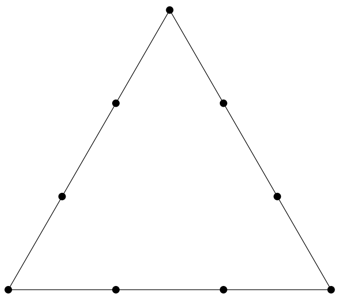

Figure 1: Triangle with two points in each edge

5. In the triangle of Figure 1, there are two points in each edge and they are obviously colinear with the vertices that bound that edge. If three colinear points do not form a triangle, how many different triangles can be formed with three of the nine points maked in the figure?

   (a) 27
   (b) 72
   (c) 84
   (d) 9!/3!
   (e) 9!

6. Each of the lights in a certain row is either blue or green, and no adjacent lights in the row are the same color. Two statements are given about these lights:

   I. There are 51 lights in the row.
   II. If the light at each end of the row were removed, there would be 24 green lights and 25 blue lights remaining in the row.

   If it is asked to determine how many green lights are in the row, one can say that

   (a) statement I *alone* is sufficient, but statement II *alone* is *not* sufficient to answer the question asked;
   (b) statement II *alone* is sufficient, but statement I *alone* is *not* sufficient to answer the question asked;
   (c) *both* statements I and II *together* are sufficient to answer the question asked, but *neither* statement *alone* is sufficient;
   (d) *each* statement *alone* is sufficient to answer the question asked;
   (e) statements I and II *together* are *not* sufficient to answer the question asked, and additional data specific to the problem are needed.

7. There are six sticks which will be used to form the edges of a tetrahedron. These sticks have the same lenght but different colors. Two arrangements of the sticks are considered to be equal if a 3-D rotation may put all the sticks of the same color in the same direction. How many different arrangements of the six sticks can be formed?

   (a) 4
   (b) 6
   (c) 8
   (d) 24
   (e) 30

8. Three machines, A, B, and C, are each capable of completing a certain production order. About their productivity, two statements are given:

   I. A alone would complete the entire order in 20 hours.

   II. B alone would complete the entire order in half the time it would take C alone to complete it.

   If it is asked the total number of hours required for the three machines, running simultaneously but independently, to complete the production order, one can say that

   (a) statement I *alone* is sufficient, but statement II *alone* is *not* sufficient to answer the question asked;
   (b) statement II *alone* is sufficient, but statement I *alone* is *not* sufficient to answer the question asked;
   (c) *both* statements I and II *together* are sufficient to answer the question asked, but *neither* statement *alone* is sufficient;
   (d) *each* statement *alone* is sufficient to answer the question asked;
   (e) statements I and II *together* are *not* sufficient to answer the question asked, and additional data specific to the problem are needed.

9. The double argument formulas of hyperbolic functions are

$$\begin{aligned} \sinh(2x) &= 2\sinh(x)\cosh(x) \\ \cosh(2x) &= 2\cosh^2(x) - 1. \end{aligned}$$

   Using the formulas above, one can say that $\dfrac{\left[1+\tanh\left(\frac{x}{2}\right)\right]\left[1-\tanh\left(\frac{x}{2}\right)\right]+2\tanh\left(\frac{x}{2}\right)}{\left[1+\tanh\left(\frac{x}{2}\right)\right]\left[1-\tanh\left(\frac{x}{2}\right)\right]-2\tanh\left(\frac{x}{2}\right)}$ is equal to

   (a) $\tanh^2(x)$
   (b) $\cosh^2(x) - \sinh^2(x)$
   (c) $\dfrac{\cosh(x)+\sinh(x)}{\cosh(x)-\sinh(x)}$
   (d) $\dfrac{1+\sinh(x)}{1-\sinh(x)}$
   (e) $\dfrac{1+\cosh(x)}{1-\cosh(x)}$

10. About the number 1003003001, one can say that

   (a) it is not divisible by 7
   (b) it is not divisible by 11
   (c) it is not a perfect square
   (d) it is not a perfect cube
   (e) all the options above are wrong

11. If n is an integer, one *cannot* say that

   (a) $\frac{1}{2}(n^2 + 3n)$ will always be an integer
   (b) $\frac{1}{3}(n^3 + 3n^2 + 2n)$ will always be an integer
   (c) $n^4 - 4n^3 + 6n^2 - 4n + 1$ will always be a perfect square
   (d) $\dfrac{-7n^3 + 27n^2 - 18n}{2}$ will always be a perfect cube
   (e) $\sqrt[3]{8n^3 + 12n^2 + 6n + 1}$ will always be an integer

12. Given

$$a = \frac{x+y}{2} \quad b = \frac{x-y}{2} \quad a' = \frac{x+2y}{3} \quad b' = x - y$$

   what is the value of $\dfrac{b'}{a'}$?

   (a) $\dfrac{2b}{a}$
   (b) $\dfrac{3a}{b}$
   (c) $\dfrac{3a-b}{2b}$
   (d) $\dfrac{3b}{2a+b}$
   (e) $\dfrac{6b}{3a-b}$

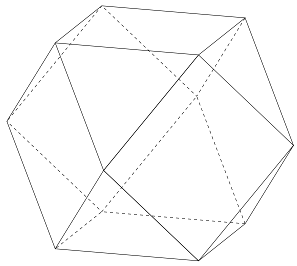

Figure 2: cuboctahedron

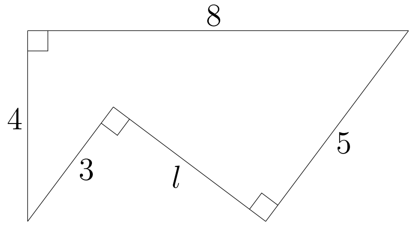

Figure 3: Nonregular pentagon

13. Figure 2 shows a cuboctahedron, which is a semiregular polyhedron that has regular squares and regular triangles as its faces. If the edge of these squares and triangles have a length of l, the volume of the cuboctahedron will be

    (a) $\frac{5}{3}\sqrt{2}\,l^3$
    (b) $\frac{7}{3}\sqrt{2}\,l^3$
    (c) $\left(\frac{2}{3}\sqrt{3}+\sqrt{2}\right)l^3$
    (d) $\left(\frac{\sqrt{2}}{3}+\sqrt{3}\right)l^3$
    (e) $\frac{1}{3}\left(5\sqrt{3}-\sqrt{2}\right)l^3$

14. In figure 3, what is the length l?

    (a) 4
    (b) $\sqrt{17}$
    (c) $\sqrt{18}$
    (d) $2\sqrt{5}$
    (e) More information is necessary in order to determine l

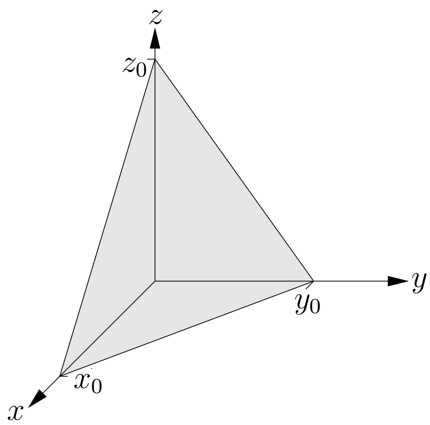

Figure 4: Plane determined by its intercepts with the coordinates axes

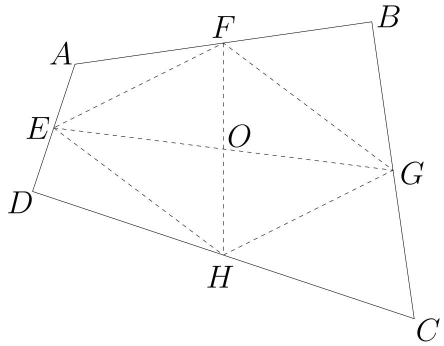

Figure 5: Quadrilateral

15. Figure 4 shows a plane which is determined by its intercepts with the coordinanates axes. The least distance between this plane and origin is calculated as

    (a) $\dfrac{\sqrt{3}}{\dfrac{1}{x_0} + \dfrac{1}{y_0} + \dfrac{1}{z_0}}$

    (b) $\dfrac{x_0\,y_0\,z_0}{x_0\,y_0 + y_0\,z_0 + x_0\,z_0}\sqrt{3}$

    (c) $\dfrac{1}{3}\sqrt{x_0^2 + y_0^2 + z_0^2}$

    (d) $\dfrac{1}{3}\sqrt{x_0\,y_0 + y_0\,z_0 + x_0\,z_0}$

    (e) $\dfrac{x_0\,y_0\,z_0}{\sqrt{x_0^2\,y_0^2 + y_0^2\,z_0^2 + x_0^2\,z_0^2}}$

16. Figure 5 shows a quadrilateral $ABCD$ together with the midpoints of its sides E, F, G and H and the intersection O of EG and FH. Mark the *wrong* option:

    (a) The area of triangle EOF is equal to the area of triangle FOG

    (b) $FO = OH$

    (c) $EF = GH$

    (d) $FH^2 + EG^2 = 2\left(EF^2 + EH^2\right)$

    (e) $\widehat{FGO} = \widehat{OGH}$